factory engine-room, at a speed increased to 195 revolutions per minute. It developed frequently 50 brake horse-power with coal gas for several hours together. Since then the engine has been taken to Cambridge, and is now engaged in regular service with a suction-producer, driving the workshops, and producing electric current for the engineering laboratory. It is left to itself like an ordinary gas-engine, giving no trouble at all, and has been in regular work for two years, the total time of running being 5000 hours.

Judging from the success which has so far been obtained, it seems likely that Prof. Hopkinson's method of cooling the cylinder will revolutionise the design and construction of large gas-engine cylinders.

RECENT PAPERS ON VERTEBRATE PALÆONTOLOGY.

A VERY remarkable announcement is made by Mr. J. W. Gidley in vol. lx., No. 27, of the Smithsonian Miscellaneous Collections, namely that an associated series of five upper cheek-teeth of a large ruminant from a Pleistocene cave-deposit near Cumberland, Maryland, U.S.A., indicate an antelope apparently closely related to the elands of Africa. So near, indeed, is the resemblance that the author deems himself justified in referring the fossil to the existing genus, under the name of *Taurotragus americanus*; and the plate showing these teeth alongside those of the existing *T. oryx* goes a long way in confirming his conclusion. It should have been mentioned that the present writer (see Cat. Siwalik Vert. Ind. Mus., part i., p. 1885) has provisionally referred certain teeth from the Indian Siwaliks to Taurotragus (=Oreas); and if the identification be correct, it would explain how eland might have reached America from Asia by the Bering Sea route. Mr. Gidley quotes the occurrence in the Pleistocene of Nevada of remains of certain ruminants described as Ilingoceros and Sphenophalus as corroborative evidence of the former existence of tragelaphine antelopes in America; but he omits to mention that although these genera were at first assigned to that group, they have been subsequently regarded as akin to the American family Antilocapridæ (Merriam, Bull. Dept. Geol. Univ. California, vol. vi., p. 292). If this be correct, is it quite impossible that the supposed eland represents another member of the same group?

In a second communication (*op cit.*, No. 26) Mr. Gidley records the occurrence of a toe-bone of a camel in a superficial deposit at the mouth of Old Crow River, in the Yukon Territory, in association with remains of mammoth, horse, and bison. The occurrence of the camel-bone confirms "the theory of the existence of a wide Asiatic-Alaskan land connection of comparatively recent date, which for a very considerable length of time served as a great highway for the free transmission of mammals between America and the Old World."

As being only in part palæontological, brief notice must suffice for a paper, by Mr. K. S. Bardenfleth, on the form of the carnassial tooth in Carnivora, published in *Vidensk. Meddel. Dansk. naturh. Foren*, vol. lxv., pp. 67–111. After reviewing the various theories of the homology of tooth-cusps, the author proceeds to observe that in order to demonstrate that the simple reptilian tooth-cone is represented by the middle one of the three longitudinally arranged cusps of the Purbeck Triconodon, and that the tritubercular crown has been formed by rotation of the other two, indisputable evidence has yet to be furnished, "first, of the Triconodon-like forms being the ancestors of Dryolestes, &c.; second, of the supposed protocone and protoconid of these being really homologous with the median cusp of Triconodon. One can scarcely imagine how such a rotation could take place, and if Gidley is right in his interpretation of the molar cusps of Dryolestes, the rotation has not taken place, but the so-called protocone is a secondary acquirement; the true protocone is still to be sought in the central one of the three outer cusps. If this holds good the whole nomenclature and theory of Osborn falls to the ground; neither protocone nor protoconid are then identical with the reptilian cone."

Three papers, by Dr. R. Broom, form part 6 of vol. vii. of the Annals of the South African Museum, and relate to the extinct reptiles of the same country. In the first of the triad the author shows that while in Pariasaurus the digital formula is 2.3.3.4.3, in the allied Propappus it is probably 2.3.4.5.3. In the second he describes, as *Noteosaurus africanus*, a new genus allied to Mesosaurus, of which three of the known species are South African, while the fourth is Brazilian. The last paper comprises a classified list of the early Mesozoic reptiles of South Africa, which, apart from dinosaurs, crocodiles, rhynchocephalians, &c., are arranged in no fewer than nine ordinal groups, brigaded in three "superorders." R. L.

AN ALGEBRA FOR PHYSICISTS.[1]

THE principal novelties in Dr. Macfarlane's calculus are that a distinction is made between linear and cyclic successions of vectors, and that the commutative law of addition, as well as that of multiplication, is abandoned. To express what most vectorists write $\beta+\alpha=\alpha+\beta$, Dr. Macfarlane writes $\Sigma(\beta+\alpha)=\Sigma(\alpha+\beta)$. Thus $\alpha+\beta-\alpha$ is not the same as β, but either three sides of a parallelogram, or three coinitial vectors, according as we take linear or cyclical succession. By introducing some subsidiary and rather artificial rules, the author is able to get formulæ that are, in appearance, analogous to the binomial and exponential theorems, and so on.

The actual divergence from quaternion results is not very great, as may be easily shown by an example. Let x be a scalar, α a unit vector, and let $\exp(x\alpha)$ be defined to mean $\Sigma(x\alpha)^n/n!$. Then $\exp(x\alpha)=\cos x+\alpha\sin x$, and if y is another scalar, $\exp(x\alpha).\exp(y\alpha)=\exp(y\alpha)\exp(x\alpha)=\exp\{(x+y)\alpha\}=\cos(x+y)+\alpha\sin(x+y)$.

But, if β is another unit vector,

$$\exp(x\alpha)\exp(y\beta)=\cos x\cos y+\alpha\sin x\cos y+\beta\cos x\sin y+\alpha\beta\sin x\sin y,$$

which differs from $\exp(y\beta).\exp(x\alpha)$, while both, in general, differ from $\exp(x\alpha+y\beta)$: the latter, observe, being by definition the same as $\exp(y\beta+x\alpha)$. Dr. Macfarlane, after writing down his exponential formula, breaks it up into four parts, practically the same as the four given by the quaternion formula above, when written in the form—

$$\exp(x\alpha)\exp(y\beta)=(\cos x\cos y+\sin x\sin y\,S\alpha\beta)+\alpha\sin x\cos y+\beta\cos x\sin y+V\alpha\beta.\sin x\sin y.$$

It must be left to physicists themselves to decide whether Dr. Macfarlane's new algebra is superior to those already available; the need of a sign to express a resultant is a rather severe handicap. To the pure analyst it presents the appearance of a conglomerate, though possibly, with a change of notation, it could be fitted into a place in the family of linear associative algebras. One thing ought to be said: it is not, properly speaking, an "extension" of quaternions. Analytically, the calculus of quaternions is a linear algebra of a perfectly definite type,

[1] (1) "Account of Researches in the Algebra of Physics," I.–III. (Reprint from Journ. Wash. Ac. of Sc., 1912.)
(2) "On Vector-analysis as Generalised Algebra" (Intern. Congress of Mathematicians, 1912.) By Dr. A. Macfarlane.