

physical features of the planets and moon. As a geologist the author claims to have formed definite views of his own on these questions, differing in many respects from commonly accepted theories; but, as he points out, it would be impossible for a writer to substantiate these varied theories unless he had travelled all over the world, besides being, at the same time, a mathematician, a physicist, a chemist, an astronomer, and a geologist. Considerable attention is given to theories of the displacement of the earth's axis.

A collection of theories of this kind, if thus propounded in a proper spirit, is not only interesting, but it opens up useful material for future discussion. On the other hand, not the least important feature is the insight which the book affords the general reader of known physical facts and phenomena connected with the earth and planets.

*A Manual of School Hygiene.* By Prof. E. W. Hope, E. A. Browne, and Prof. C. S. Sherrington. New and Revised Edition. Pp. xii+311. (Cambridge University Press, 1913.) Price 4s. 6d.

THE first edition of this manual, which was reviewed in our issue for August 15, 1901 (vol. lxiv., p. 373), was reprinted on three occasions before the appearance of the book in its present form. Six chapters on physiology by Prof. Sherrington have here been added. They aim at emphasising the salient portions of the subject, and deal with the body considered as a mechanism, the blood and its circulation, respiration, food and digestion, the temperature of the body, and muscle and nerve.

*Library Cataloguing.* By J. Henry Quinn. Pp. viii+256. (London: Truslove and Hanson, Ltd., 1913.)

MR. QUINN'S book should prove of real service as a guide for young librarians to the various codes of cataloguing rules. His bright, helpful chapters should certainly convince the beginner in library work that the office of librarian is no sinecure; and the arrangement of his matter, and the subjects chosen for treatment, should enable information on practical cataloguing to be obtained with a minimum expenditure of trouble.

LETTERS TO THE EDITOR.

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Energy in Planetary Motions.

IF a particle of mass  $m$  be brought from infinity to distance  $r$  by the action of a central attraction varying as the inverse square of the distance, the potential energy exhausted in the process is  $m\mu/r$ , where  $\mu$  is the "intensity of the centre." If the particle has experienced no resistance to its motion the kinetic energy is given by the equation

$$\frac{1}{2}mv^2 = m\frac{\mu}{r}$$

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But if the particle be made to move in a circle of radius  $r$  about the centre of force, the speed  $v$  is given by

$$v^2 = \frac{\mu}{r}$$

and the kinetic energy  $\frac{1}{2}mv^2$  represents only half the potential energy exhausted. The other half must have been dissipated or disposed of in some way or other.

Similarly, if the particle be brought in from motion in a circle of radius  $r'$  about the centre to motion in the circle of radius  $r$ , so that potential energy of amount  $m\mu(1/r - 1/r')$  is exhausted, the kinetic energy has been increased by only  $\frac{1}{2}m\mu(1/r - 1/r')$ , so that again only half of the potential energy exhausted is represented by the orbital motion, and the remainder has been expended in doing work against resistance of some sort. The central force has, in fact, done exactly twice as much work as that represented in the increase of the kinetic energy.

All this, of course, is perfectly elementary and well known, but it is nevertheless a curious dynamical fact that exactly half of the work done by the attraction must be expended in overcoming resistance.

I have not seen the corresponding theorem in elliptic motion anywhere explicitly stated. It is as follows:—*The time-average of the kinetic energy, taken for one revolution in the orbit, is half of the corresponding time-average of the potential energy exhausted in the passage from infinity to the distance  $r$ .* A similar theorem holds, of course, for the differences of energy concerned when the particle is transferred from one orbit to another about the same centre.

Let  $2a$  be the length of the major axis of the elliptic orbit. The speed  $v$  at distance  $r$  from the centre is then given by

$$\frac{1}{2}v^2 = \mu \left( \frac{1}{r} - \frac{1}{2a} \right),$$

which, multiplied by  $m$ , is the equation of energy. The potential energy exhausted from infinity to distance  $r$  is again  $m\mu/r$ , and it can easily be shown that the time-average of the kinetic energy in the orbit is  $m\mu/2a$ .

Parenthetically, it may be remarked that this result is most easily and elegantly established by the following Newtonian process. If when  $r$  is the distance of the particle from the centre of force (one focus of the ellipse)  $r'$  be the distance from the other focus, and  $p, p'$  be the lengths of the perpendiculars from the foci on the line of motion at the instant, we have  $r/r' = p/p'$ , and, therefore, since  $pp' = b^2$ , where  $b$  is the length of the semi-minor axis, we have  $r'/r = p'^2/b^2$ . But the equation for  $v^2$  can be written

$$v^2 = \frac{\mu}{a} \frac{2a - r}{r} = \frac{\mu}{a} \frac{r'}{r}$$

Hence integrating for a period of revolution  $T$  we get

$$\int_0^T v^2 dt = \int v ds = \frac{T}{b} \sqrt{\frac{\mu}{a}} \int p' ds,$$

where  $ds$  is an element of the path, and the integrals with respect to  $s$  are taken once round the ellipse. Now, clearly  $\int p' ds$  is twice the area of the ellipse—that is,  $2\pi ab$ . Thus

$$\frac{1}{2}m \int_0^T v^2 dt = \pi m \sqrt{\mu a}.$$

The period  $T$  is  $2\pi \sqrt{a^3/\mu}$ , and so the mean kinetic energy is