side of the bevel gears at those parts which were in mesh at the moment the error occurred, and the error was thereby reduced to about eight-tenths of a second.

The sun was photographed on 148 days, and showed spots on 88 days. In future, the "Nautical Almanac" publications are to be stored and distributed by the Naval Observatory librarian.

EPHEMERIS FOR BORRELLY'S COMET, 1911*e*.—To No. 4572 of the *Astronomische Nachrichten*, M. Schaumasse contributes an ephemeris for comet 1911*e*, which is at present about a degree north of 36 Lyncis, and is travelling in the direction of β Leonis Minoris. This comet is extremely faint, but an observation by M. Schaumasse, with the Nice equatorial *coudé* on April 19, showed that the error of the ephemeris was only −3s., 0′.

THE TEACHING OF MATHEMATICS.[1]

The Content of the School Course in Mathematics.

A SYSTEM of education designed on broad lines to prepare pupils for some particular occupation is not only the best training for that particular occupation, but it is better as a "general education" than a system which has been designed simply as a general education, and not as a preparation for any particular calling. For a boy willingly undertakes work which clearly leads up to the solution of a real and interesting problem, even if that problem is one that belongs to his neighbour's after-life and not to his own. But the course designed for "general education" tends to become a "mental discipline" lacking in interest, and such discipline deadens the mind and makes the boy a machine.

In Papers Nos. 15 and 16 of this series, Mr. Carson and Mr. Durell advocate the inclusion in a school course of certain methods of great beauty, which to a few boys will be a source of delight. But the authors of those papers have no criterion of the suitability of these subjects beyond their own love of them. To a certain point that is a true criterion; what has given pleasure to one person has a good chance of giving pleasure to another; and all the subjects which they advocate deserve a place in a system of recreations for the mathematician's leisure hour. But to determine which of these methods and subjects are to be thrust upon every boy of an ordinary degree of mathematical ability, some better criterion is necessary. I do not say that I would exclude any of these methods, but only that they have not yet been judged on a suitable criterion.

That suitable criterion must be a consideration of the needs in after-life of certain groups of boys. In many cases mathematics is a form of technical knowledge required for the after-career, *e.g.*, for the careers of engineer, mathematical schoolmaster, professor of mathematics. In such cases the content of the subject will be determined by a wide interpretation of the requirements of the career, the treatment of the subject being of the broadest and every problem viewed from many points of view. The boy to whom mathematics is merely a part of his general education will, so far as he goes, study along with the technical group with which he has most in common. It is not necessary that each boy's future career should be planned in advance; all boys, technical, semi-technical, and non-technical, will study together for a time; then gradually the non-technical boys will drop out, and the remainder will bifurcate according to their varying intellectual powers and their varying technical needs.

These are the views to which observation, experiment and reflection are leading students of education. Many a doubter will be converted by a study of Mr. Mercer's admirable account of the teaching at the naval colleges (Paper 17). It is a document which every mathematical master should have by him. Some small portions of the course are special to the requirements of the Navy, but the course as a whole makes an excellent starting point from which to lay out a scheme for any school.

In Paper No. 12 Mr. Usherwood provides further evidence in favour of our principles. The close correlation of mathematics with engineering has given his boys a breadth of mathematical knowledge and a real grasp such as would have been incredible a generation ago. Mr. Usherwood justifies his procedure by quoting Mr. Branford's classification of the impulses which urge towards mathematical study, a classification also held by Dr. Nunn. Of these impulses, the utilitarian is the chief one at the school stage, and every central truth should be made to arise in response to some demand arising from a practical problem. Mr. Usherwood holds that manual as well as mental dexterity should be involved in the practical problem from which an investigation sets out, and he petitions for a greater place in the curriculum for suitable manual training.

Further support to the principles enunciated above is given by Mr. Palmer's historical account of the teaching of arithmetic. It is an excellent account of the changes which have been made in the last quarter-century. A generation ago "general education" was the cry, and if any method had a "bread-and-butter" value that was sufficient reason for its exclusion. The course consequently contained such monstrosities as "true discount." The true criterion has now been adopted; in part, half unconsciously. More conscious application of the criterion will in time recognise that most fractions should be dealt with in decimal form, and will greatly reduce the time spent on vulgar fractions, greatest common factor, and least common multiple. We learn from Mr. Palmer how far removed the school treatment of stocks and shares is from business practice. The whole subject seems to us unsuited to the school. The difficulty lies in the realisation of the circumstances of the problem; the circumstances are far removed from a boy's experience, and the explanation of them profits him nothing. The circumstances once realised, the arithmetic is child's play.

The Methods of Mathematical Study.

The various methods of mathematical investigation have been added one by one at various times to our available stock of tools. On the historical principle that the development of the individual should copy the history of the race, it is appropriate that these various tools should be put in the pupil's hand in the order of their discovery. It is, however, the practice to follow the development of the race too closely, and to discuss by the more primitive method all the problems for which our ancestors used it, regardless of the fact that a later method is a more suitable weapon with which to attack many of these problems. Such exactness of recapitulation cannot be justified; it is the haphazard result of the successive

[1] Papers on the Teaching of Mathematics in the United Kingdom, published by the Board of Education:—
(12) "Mathematics with relation to Engineering Work in Schools." By T. S. Usherwood. (1912.) Price 2*d*.
(13) "The Teaching of Arithmetic in Secondary Schools." By G. W. Palmer. (1912). Price 2½*d*.
(14) "Examinations for Mathematical Scholarships." By Dr. F. S. Macaulay and W. J. Greenstreet. (1912). Price 3*d*.
(15) "The Educational Value of Geometry." By G. St. L. Carson. (1912.) Price 1½*d*.
(16) "A School Course in Advanced Geometry." By C. V. Durell. (1912.) Price 1½*d*.
(17) "Mathematics at Osborne and Dartmouth." By J. W. Mercer and C. E. Ashford. (1912.) Price 2½*d*.
Earlier papers were noticed in NATURE of March 14.

origins of the various methods; the physiologist from whom education has borrowed the historical principle says that "the history of the individual is a *blurred* recapitulation of the history of the race."

Too exact a recapitulation is wasteful of time and deadens the intellect. The recapitulation must be a blurred one; the barriers between the various branches of mathematics must be broken down, and the pupil given freedom to select for any problem whatever tool he finds most appropriate.

In Paper 16 Mr. Durell drives home this principle. The freedom to treat a problem by Euclid's method, by Descartes', or by Monge's, by the principle of duality or by that of continuity, gives to the pupil a breadth of view and to the subject a unity otherwise unattainable. It reduces the multitude of properties of geometrical figures to a small number of greater generalisations which the mind can carry without effort. And it effects a saving of time, which makes possible a much further advance in mathematics than is now customary.

Mr. Durell rightly reduces to small compass the Euclidean treatment of conics, but he retains conics as the chief material to which the various methods are to be applied. His course might be further improved by the substitution in some cases of other material, such as an occasional higher algebraic curve, a transcendental curve, or a surface.

The Postulates of Geometry.

Mr. Carson (Paper 15) pleads for more system in the treatment of elementary geometry, in order that the pupil may gain a better grasp of the subject and have time to pursue his studies further. Mr. Carson would assume as postulates all the geometrical properties which can be looked upon as "intuitive," and build a system of reasoned geometry upon these; a suggestion which deserves serious consideration. The elaboration of this idea must involve some preliminary discussion of the nature of intuition. Intuition varies greatly from individual to individual; that "things equal to the same thing are equal to one another" is not an intuition to every child (see Branford's "Principles of Mathematical Education"); and, on the other hand, to an occasional genius results are intuitive which involve prolonged investigation for the average mathematician. Intuitions depend upon experience, and differ according to the experience of the individual.

It will clearly be necessary to give precision to each particular property which is to be assumed as an intuition. One valuable method of giving such precision is strangely repugnant to Mr. Carson, I mean that of numerical illustration. This method has real value, not only for these intuitions, but also for ensuring the comprehension of a property of which the proof is to follow. Nevertheless, when worked out Mr. Carson's scheme would probably differ little from some courses now in use.

Mr. Carson's main thesis is that if the inclusion of mathematics in the school curriculum is to be upheld, its study must be justified as an end in itself, and not by any consideration of utility. This view is best judged by the conclusions to which it leads him. One such conclusion is that the study is essential for girls as well as for boys; perhaps if Miss Burstall's excellent discussion of that topic in a recent number of *The Mathematical Gazette* had been available at the time when Mr. Carson wrote this paper, he might have modified his views.

We have already referred to Mr. Carson's criterion of the content of the mathematical course—"mathematics for its own sake." To most of us beauty is closely connected with utility; there are on the high road of progress just as many and as lovely views to be seen as in Mr. Carson's bypaths. For many of us, also, the high road provides bread and butter along with beauty; at the present day the view is all too prevalent that real work and beauty are incompatible.

But really Mr. Carson is barely half in earnest. He is constantly falling into some utilitarian justification for his teaching, and then pulling himself up short. And the programme he sketches is excellent, chiefly because he keeps so close to the concrete and to utility.

Examinations.

In recent years there has been much discussion of the value of literary examinations, some holding them to be the only true criterion of a pupil's ability, others holding them entirely harmful. The truth would appear to lie between these extremes. On the one hand, no literary examination can tell us much of the character of a boy, and there are subjects in which training is the great element, and knowledge so small an element that any attempt to examine would spoil the value of the subject. There are, on the other hand, many subjects in which examination has real value—provided it is properly conducted.

An examining body cannot escape the responsibility of influencing schools, whether for good or ill. If the examiner is ignorant of the schools his influence will be bad; he must in some way be put in close touch with the school. He must also not be a mere hack, but have a fresh interest in the subject and some knowledge of educational principles. With that granted, there is ground for hope that his influence on the schools will be good. Another thing of much value is difficult to get, namely, the criticism of the business man who has no expert knowledge of the subject but a real knowledge of the kind of boy he wants in his business. I remember Prof. Henrici's modest account of his early mathematical development as teacher in a technical college. The business committee wanted certain things done which seemed impossible to the young professor with his academic views. But he agreed to try, and speedily he concluded that the business men had been perfectly right.

Messrs. Macaulay and Greenstreet (Paper 14) discuss the scholarship examinations on which the universities select entrance scholars. The discussion concerns Cambridge chiefly, and the authors make a strong case for their view that the universities are not sufficiently acquainted with the conditions of the schools and that more weight should be attached to the opinions of the schoolmasters who prepare the boys for the examinations. The authors deserve all sympathy in their desire that pupils should not waste time in exploring bypaths and in the acquisition of excessive skill in manipulation, but should push on along the main road. Some of their suggestions, however, scarcely carry conviction. Consider, for instance, their disapproval of the graphical method in statics, a method of such value for giving a grasp of principle. Take, again, their view that a boy should sit still and watch his master draw algebraic graphs without drawing them himself.

DAVID BEVERIDGE MAIR.

BIOLOGICAL PAPERS FROM PRAGUE.

PROF. HLAVA (Bull. Internat. Acad. Sci., Prague, xv. Ann.) has found, in the blood of children infected with measles, oval or rod-like bodies, which he regards as probably of protozoan nature. In a blood-smear from another infected child (who also exhibited severe anæmia due to the presence of numerous whip-worms in the intestine),