ancient temples. Finally, Sir Robert Hadfield's account of "Sinhalese Iron and Steel of Ancient Origin" throws an interesting light on the materials and methods in Ceylon many centuries ago, particularaly in the production of steel tools and implements. We hope to give a separate abstract of this paper in a later issue.

The exceptionally full and interesting programme of the meeting is completed by a series of valuable papers dealing more directly with steel manufacture, including an historical survey of forty years' progress of the industry by the president (Mr. Arthur Cooper) in his address, an interesting paper on steam engines for driving rolling-mills by Mr. J. W. Hall, and an account of the Nathusius electric steel furnace by its originator, with several other contributions of a similar character. Altogether the institute is to be congratulated upon a singularly successful meeting, which revives the traditions of the best days of its history.

M. POINCARÉ'S LECTURES AT THE UNIVERSITY OF LONDON.

I.—May 3.—*The Logic of the Infinite.*—Some years ago, M. Poincaré said, he had published a certain number of articles upon the subject, which had involved him in a veritable polemic. He would not attempt to renew the arguments that had been used on either side, or to bring forward any fresh arguments, as he believed that the divergence of the two schools was irreducible. It arose from an essential difference of mentality; he would therefore accept it as an experimental fact, and would endeavour to account for this divergence. For the first school, whom, for the sake of convenience, he would call Pragmatists, the infinite was derived from the finite; for the second, the Cantorians, the infinite pre-existed, and the finite was only a small piece of the infinite. From another point of view, to use the language of the scholastics, the Pragmatists were *extensionists,* while the Cantorians were *comprehensionists.* This appeared in the nature of the definitions used by the two schools. For the first a definition consisted in the addition of one new object, expressed in terms of the aggregate of known objects; for the second a definition was a fresh subdivision of the aggregate of all objects known and unknown. The Pragmatists were idealists, and for them an object did not exist until it had been *thought.* The Cantorians were realists for whom the existence of objects was independent of a thinking subject. For them the infinite was independent of man or any thinking being; it was pre-existent and was discovered by man.

II.—May 4.—*Time and Space.*—The conception of space arose from our muscular sense. When we saw an object we knew the movements necessary to attain it. The idea of space, then, was the association between certain sensations and certain movements. To the whole of space the principle of relativity applied, that is to say, we had no means of perceiving a transportation, a magnification, or a deformation of the universe, provided that in the transformation all objects were subject to the same law. Space, in fact, was "soft and without rigidity." We appreciated the relations between objects in space by means of our instruments of measurement, of which our body was one, and the science of geometry was a study of these instruments. But the instruments were not perfect, and therefore we replaced them by a series of ideal instruments for the purposes of our geometry, which thus depended upon an aggregate of conventions approximating to the actual laws, but simpler. The principle of relativity also applied to time; if all actions were retarded uniformly we had no means of perceiving it.

A revolution had recently been brought about by the researches of modern physicists, especially those of Lorenz. Formerly the action of one body upon another was supposed to be instantaneous. But if we supposed that such an action was transmitted through the intervening space at a finite speed, the question of priority of action became very difficult. Formerly we had considered an action α to be anterior to a dependent action β, when α could be regarded as the cause of β. But in the new mechanics, if β occurred too soon it might happen that α could not be regarded as the cause of β, nor β as the cause of α. It might be necessary at this stage to abandon our former mechanical conventions and to adopt new ones.

III.—May 10.—*Arithmetical Invariants.*—If an algebraic form, in two variables, say, $F(x, y)$, was subjected to the transformation

$$x \rightarrow \alpha x+\beta y,\ y \rightarrow \gamma x+\delta y, \text{ where } \alpha\delta-\beta\gamma=1 \quad . \quad . \quad (1)$$

there were certain functions of the coefficients of F which remained unchanged. These were algebraic invariants. Suppose now that α, β, γ, δ, and also the coefficients of $F(x, y)$ were restricted to be whole numbers, positive or negative, $F(x, y)$ would possess the same invariants as before, but it would also possess others which were termed *arithmetical invariants.*

The simplest form was $F(x, y)=ax+by$. This form possessed no algebraic invariants. Some arithmetical invariants, however, could be obtained which were related to the Weierstrassian elliptic functions, the thetafuchsian functions, and the functions of Jacobi.

In the case of quadratic forms it was necessary to distinguish between the definite and the indefinite. The definite quadratic form might be reduced for this purpose to a pair of linear forms, but for an indefinite form invariants could only be found if we took certain subgroups of the group of transformations considered instead of the group itself.

IV.—May 11.—*The Theory of Radiation.*—Planck had enunciated some ideas, which, if they were accepted, would bring about in the science of physics the most profound revolution that had occurred since the time of Newton. We owed to Newton the principle that the laws of nature could be expressed in the form of differential equations. According to Planck, phenomena satisfy not differential, but finite difference equations.

By the method of statistics applied to a very great number of separate molecules we arrived at one of the fundamental theorems of thermodynamics, that of Maxwell on the equipartition of mean kinetic energy. Upon the same basis we arrived at Wien's law of radiation and Rayleigh's law. The last was consistent with the theorem of Maxwell, but it was not justified by experiment.

Planck supposed that there existed in incandescent bodies a very great number of *resonators,* each corresponding to a certain wave-length of light; these resonators could only acquire or emit energy by a definite increment: a *quantum* or atom of energy. Planck obtained in this way a law of radiation, which was justified by experiment, but which was not consistent with Maxwell's theorem. M. Poincaré found that if, instead of considering the action of light upon a molecule, we applied the ideas of Planck to the action of a molecule upon light, we should be forced to conclude that diffusion took place with a certain retardation, and this was certainly not true. Thus the hypothesis of Planck was unsatisfactory, and no solution to the problem was at present in sight.