

the addition of granaries for dried seeds—the different sorts kept separately—and with more blind passages. In place of the leaves with which a mole fills its more roughly constructed inner sanctuary, the kangaroo rat's nest was lined with "a thick felting of fine grass and weed silk, and, inside all, a lining of softest feathers." "I think," he writes, "that every gay little bird on the plains must have contributed one of its finest feathers to that nest."

Among the best passages in the book are those in which Mr. Thompson in his first chapter reads the records of the old ram's long life in the gravings of his horns. The deep dent tells of the early battle in which he won his spurs. The two dark-coloured, wrinkled rings close together lower down are reminders of the years of starvation and the sickness which carried off the weaker members of the flock, and the bolder ridges wide apart recall the prosperous years that followed.

He has much to tell that is worth learning, and, left to himself, can tell it excellently. It will be a misfortune to many lovers of natural history besides himself if Mr. Thompson is beguiled into sacrificing himself on the shrine of the admirers who, as he tells in his preface, "bitterly denounced" him for confessing that in unregenerate days he was not above killing a dangerous wolf when he could.

T. D. P.

#### CUBIC AND QUARTIC CURVES.

*An Elementary Treatise on Cubic and Quartic Curves.*

By A. B. Basset, F.R.S. Pp. xvi + 255. (Cambridge: Deighton, Bell and Co., 1901.) Price 10s. 6d.

NOW that Salmon's "Higher Plane Curves" is out of print there is undoubtedly room for a good book on the subject. The purpose of such a book would be to give students who had read conic sections and the infinitesimal calculus a good knowledge of the main lines on which the theory of curves has been developed. The bookwork would contain discussions of the chief theorems; those of less importance would be given as examples, and would furnish the student with abundant matter for independent thought. The proofs given would, so far as possible, be models of rigour and elegance, and in the rare cases where rigour was sacrificed for the sake of simplicity this would be confessed. The book before us has not been written altogether on these lines. There are no examples, and a great deal of space is taken up in proofs of the properties stated that could, in our opinion, have been put to better use; moreover, the proofs given are not always satisfactory, and even the theorems themselves are sometimes wrongly stated.

After two introductory chapters, chapter iii. deals with tangential coordinates, reciprocal polars and foci, chapter iv. with Plücker's equations. Then we have a chapter on "cubic curves" (pp. 56-73) and another on "special cubics" (pp. 74-96). The special curves discussed are circular cubics, and in particular some that are the inverses of conics, the semicubical and cubical parabolas, the folium of Descartes, the witch of Agnesi. Chapters viii., ix., x. are respectively on "quartic curves" (pp. 101-132), "bicircular quartics" (pp. 133-161), "special quartics" (pp. 162-204). Non-singular, or, as the author prefers to call them, anautotomic quartics,

receive attention for three pages only (115, 117, 122). The special quartics discussed are the cassinian, the lemniscates of Bernoulli and Gerono, cartesian, limaçons, the cardioid and the conchoid of Nicomedes. Chapter xi. treats of "miscellaneous curves," roulettes, the evolute of an ellipse, the involute of a circle, the catenary, tractory, elastica and spirals. Chapter xii. is on projection. Some useful references are given in footnotes.

The author has not found space for any general discussion of the forms of cubic and quartic curves, or of the expressions for the coordinates of a variable point on a curve in terms of a parameter, even when the curve is unicursal. The theory of residuation is not mentioned. The following are some of the matters of detail in which the book might be improved.

It is a good thing to "give special prominence to geometrical methods," but we do not think it is sound to estimate, say, the number of tangents that can be drawn from a cusp, real or imaginary, by inspection of the figure (p. 18), especially when no discussion of the form of a curve near a real cusp has been given; the question is in its essence an algebraical one and cannot really be decided except on algebraical grounds.

A process is given (§ 2) for finding the eliminant of two binary quantics of degree  $n$ . The result would be of the degree  $2^n$  in the coefficients.

The condition given on p. 4 for the equality of  $r$  roots of an equation would lead us to conclude that the equation  $3x^4 - 4x^3 + 1 = 0$  has three equal roots.

In the proof of Plücker's equations (chap. iv.) it is only shown that  $m$  cannot exceed  $n(n-1) - 2\delta - 3\kappa$ , and that  $\iota$  cannot exceed  $3n(n-2) - 6\delta - 8\kappa$ . It is not proved that the curve and the Hessians meet only at multiple points and points of inflexion.

On p. 62 it is proved that the node of a nodal cubic is a pole of the line of inflexions. The author must have forgotten for the moment that the node is a pole of any line whatever in the plane.

Cayley's theory of conjugate poles on the Hessian of a cubic is treated by means of trilinear coordinates; the figure on p. 70 does not altogether correspond with the text, for it is proved that  $K$  lies on the line  $PQ$  and that  $C(AMBK)$  is a harmonic pencil. Also it is surely wrong to say that when "A is given, there are in general three conjugate poles corresponding to A" (p. 71).

The proof (p. 115) that a quartic cannot have more than eight real points of inflexion is very flimsy; it consists of an appeal to an extreme limiting case.

The assumption that a ternary quartic can be put in the form  $lU^2 + mV^2 + nW^2$  is justified by counting the constants (p. 117), although later (p. 240) the reader is very rightly warned "that counting the constants is not always a safe process."

On p. 122 we have the theorem:—

"A conic can be drawn through the eight points of contact of any four double tangents to a quartic." It is well known that this is not true, and it is, in fact, inconsistent with the theorem at the foot of the same page.

There are some other points on which we do not agree with the author, but notwithstanding its drawbacks, the book contains much that is interesting and important.