By the calculus of finite differences we obtain :

$$\frac{\Delta p}{h} = \frac{p(x+h)-p(x)}{h} = a_1 + a_2 x + \frac{a_3}{2}(x+h)x + \frac{a_4}{2.3}(x+h)x(x-h) + \dots$$

and

$$\frac{\Delta^2 p}{h^2} = a_2 + a_3(x+h) + \frac{a_4}{2}(x+h)x + \frac{a_5}{2.3}(x+2h)(x+h)x + \dots$$

Therefore :

$$ha_1 = \Delta p \text{ for } x=0 \text{ and } h^2 a_2 = \Delta^2 p \text{ for } x = -h$$

By proceeding in the same way we find $h^{2v}a_{2v} = \Delta^{2v}p$ for $x = -vh$ and $h^{2v+1}a_{2v+1} = \Delta^{2v+1}p$ for $x = -vh$.

The value of $\Delta^{2v}p$ for $x = -vh$ is the number in the column for $\Delta^{2v}p$, which stands in the horizontal line corresponding to the stated value of θ (105 in our case), while the value of $\Delta^{2v+1}p$ for $x = -vh$ is the number in the next column just below this line. The mean of this number and the one above it we have before denoted by m_{2v+1}; we now add the notation m_{2v} for the value of $\Delta^{2v}p$ for $x = -vh$. As m_{2v+2} is the difference of the two numbers, whose mean is m_{2v+1}, we can write $m_{2v+1} + \frac{1}{2}m_{2v+2}$ instead of the value of $\Delta^{2v+1}p$ for $x = -vh$.

We have therefore :

$$a_{2v} = m_{2v}h^{-2v}$$

and

$$a_{2v+1} = (m_{2v+1} + \tfrac{1}{2}m_{2v+2})h^{-2v}.$$

Substituting these values in the expression for p we have :

$$p = a_0 + (m_1 + \tfrac{1}{2}m_2)\frac{x}{h} + \frac{m_2}{2}\frac{x(x-h)}{h^2} + \frac{1}{2.3}(m_3 + \tfrac{1}{2}m_4)\frac{(x+h)x(x-h)}{h^3} + \dots$$

General terms :

$$\frac{1}{2.3\ \dots\ 2v+1}(m_{2v+1} + \tfrac{1}{2}m_{2v+2})(x+vh)\dots x \dots (x-vh)h^{2v-1}$$

$$+ \frac{1}{2.3\ \dots\ 2v+2}m_{2v+2}(x+vh)\ \dots\ x\ \dots\ (x-vh)(x-(v+1))h^{-2v-2}.$$

To find the value of $\frac{dp}{d\theta}$ we now need only differentiate according to x and make x equal zero.

Thus we obtain :

$$h\frac{dp}{d\theta} = (m_1 + \tfrac{1}{2}m_2) - \frac{m_2}{2} - \frac{1}{2.3}(m_3 + \tfrac{1}{2}m_4) + \frac{1}{2.3}\frac{m_4}{2} + \dots$$

General terms :

$$\frac{(-1)^v}{2.3\ \dots\ 2v+1}(m_{2v+1} + \tfrac{1}{2}m_{2v+2})2^2 \cdot 3^2\ \dots\ v^2$$

$$+ \frac{(-1)^{v+1}}{2.3\ \dots\ 2v+2}m_{2v+2} \cdot (v+1) \cdot 2^2 \cdot 3^2\ \dots\ v^2;$$

or by contracting two consecutive terms :

$$h\frac{dp}{d\theta} = m_1 - \frac{1}{2 \cdot 3}m_3 + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5}m_5 \cdot 2^2 - \dots$$

The second differential coefficient is found in a similar way. It is only necessary to observe that the second differential coefficient of $(x+vh)\ \dots\ x\ \dots\ (x-vh)$ vanishes for $x=0$ and that of $(x+vh)\ \dots\ x\ \dots\ (x-(v+1)h)$ is equal to $2 \cdot (-1)^v \cdot 2^2 \cdot 3^2\ \dots\ v^2h^{-2v}$. Therefore we obtain

$$h^2\frac{d^2p}{d\theta^2} = m_2 - \frac{2}{2 \cdot 3 \cdot 4}m_4 + \frac{2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \cdot 2^2 \cdot m_6 - \dots$$

General term :

$$\pm\frac{2}{2 \cdot 3 \cdot 4\ \dots\ 2v+2}2^2 \cdot 3^2\ \dots\ v^2 \cdot m_{2v+2}.$$

Hannover, Technische Hochschule. C. RUNGE.

PROF. RUNGE'S proof is longer and more difficult than mine; but his result is in simpler shape, and possesses the great merit of giving the successive approximations as the terms of a regular series. J. D. EVERETT.

22 Earl's Court Square, July 28.

The So-called "Thunder"-storm.—Prevalence of Anticyclones.

IT must have occurred to others besides myself how very absurd it is to designate a meteorological phenomenon by the least important of its characteristics, viz. the noise it makes. We never speak of a hail-storm as a "rattle"-storm, or a shower of rain as a "patter"-storm; why then should we call an electrical disturbance a "thunder"-storm? Thunder, though no doubt terrifying to savages and children and old ladies (one or two of whom have, I believe, been killed by the fright of it), and though of some interest as an acoustic phenomenon, is absolutely the most trivial of the accompaniments of an electrical discharge.

It would seem hopeless to eradicate the childish term entirely from popular language, but surely in the scientific reports and forecasts issued by the Meteorological Office, and in scientific literature generally, the term "electric storm" (or disturbance) might replace "thunderstorm."

With regard to the late prevalence and persistence of anticyclonic conditions over the centre and south of our islands, I wish to suggest that it may be connected with the unusual extension southwards of the Polar ice-pack this summer. I saw it stated about a month ago that even Spitsbergen was then surrounded by ice, most of the fiords being quite inaccessible. When I was there in July 1896 we could only just see the blink of the pack in the north horizon.

Now, it is an ascertained and easily intelligible fact that areas of cold (water or ice) on the earth's surface have a tendency to cause the formation of areas of high pressure or dense air in the atmosphere above them. The result would be, not only a prevalence of anticyclones in high latitudes over the North Atlantic, but also the persistent extension of the northern edge of the great "Atlantic anticyclone" over the south and centre of England (attracted, as it were, by the high pressure in the north); so that cyclones which usually strike the south-west of Ireland or the coast of Cornwall have been "fended off" to the north of Scotland, with the result of heat and drought over England.

I only put this forward as a suggestion, and I should be glad if any of your Icelandic or Norwegian readers would supply details of the position of the Polar ice-pack, temperature of the sea in the North Atlantic, &c., for I have learnt to mistrust all statements appearing in those interesting, and often sensational, works of fiction—the daily papers. METEOR.

August 12.

Scoring at Rifle Matches.

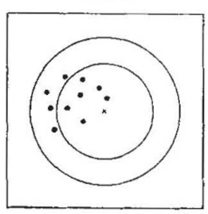
WHILE the Bisley meeting is still fresh in the memory of those interested in rifle shooting, it seems worth while to call attention to the rather unsatisfactory nature of the method of scoring now in general use.

What brings the matter into special prominence is the large number of "best possibles" always made in recent years.

With a satisfactory system of scoring such a phrase ought only to apply when every shot passes through the same hole in the centre of the bull's-eye.

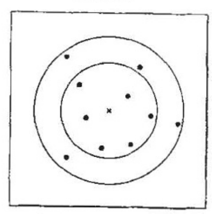
The present practice, however, gives the same number of marks to shooting of widely differing merit, and this must always be the case as long as the result is made to depend on the distance of each shot from the centre of the target, irrespective of the distance of the shots from one another (see Figs. 1 and 2).

FIG. 1.



Ordinary score 46.
By moment of inertia 24·5.

FIG. 2.



Ordinary score 46.
By moment of inertia 18.

The merit of any series of shots really depends on two elements, namely, the distance of the average direction of the whole series from the centre of the target and the compactness with which the individual shots are grouped about that direction.

The importance to be assigned to each of these elements may vary with the object for which the shooting is undertaken, but a knowledge of both is essential in estimating its quality.

If the object be to get all the shot as near the centre of the target as may be, the same importance should be attached to close grouping as to the mean direction, as will be shown further on.

Since any practical method of scoring must be rapid and easily understood by people who are not mathematical, it would probably be asking too much if it were proposed to treat each result in an accurate manner, simple though the required arithmetic is; but some modification of the accurate method could probably be made sufficiently simple for general use, which would give a much truer estimate of the goodness of the shooting than that now in use.

The accurate plan of estimating the value of any series of shots consists, in mathematical language, of finding the distance of the centre of gravity of the group from the centre of the target, and taking the radius of gyration of the group about its centre of gravity. The goodness of the shooting will then be measured by the reciprocal of the sum of the squares of these quantities, each multiplied by a constant, and it will presently be shown that if, as in an ordinary match, the object is to hit the centre of the target, these constants are equal.

A convenient way of finding the centre of gravity and radius of gyration is to have the target divided into 100 squares by eleven vertical and eleven horizontal lines (see Fig. 4), the position of the shot being recorded by naming the square through which it passes; (for instance, a shot in the fourth vertical row and fifth horizontal row would be recorded as 4·5).

If we call the number of the vertical row x, and the number of the horizontal row y, very simple algebra will prove—in fact, it is obvious—that for a series of n shots the coordinates (h . k) of square containing the centre of gravity of the shots will be

$$h = \Sigma x/n \quad k = \Sigma y/n,$$

where Σx denotes the sum of all the x's, and Σy denotes the sum of all the y's.

Since the score does not record the position of an individual shot with greater accuracy than the width of a single square this is equivalent to the assumption that each shot passes through the centre of the square it hits, and that the origin of the coordinates is in the centre of the square at 0 (off the target) (see Fig. 4).

Thus the coordinates of the centre of the target will be

$$x = 5{\cdot}5 \quad y = 5{\cdot}5,$$

and the distance of the centre of gravity from the centre of the target is

$$R = \sqrt{(\Sigma x/n - 5{\cdot}5)^2 + (\Sigma y/n - 5{\cdot}5)^2}.$$

The radius of gyration of the group about an axis normal to the plane of the target, and passing through the centre of gravity is

$$\rho = \sqrt{\Sigma x^2/n - h^2 + \Sigma y^2/n - k^2}.$$

To examine the relative importance of the closeness of the shots to one another and the distance of their mean from the centre of gravity, consider the effect of slightly varying each of the quantities R and ρ.

The question to be answered is: "If of two groups one is represented by R and ρ, and the other by R $+ d$R and $\rho - d\rho$, which gives evidence of the best shooting?"

In Fig. 3 let C be the centre of the target, and G the centre of gravity of the group, and P Q R a circle described with radius ρ about G, so that C G = R, G P = ρ; then if C G P = θ, the distance (r) of P from C is

$$r = \sqrt{R^2 - 2R\rho\cos\theta + \rho^2},$$

differentiating with respect to R and ρ, we have

$$\frac{dr}{dR}\bigg/\frac{dr}{d\rho} = -\frac{\rho - R\cos\theta}{R - \rho\cos\theta};$$

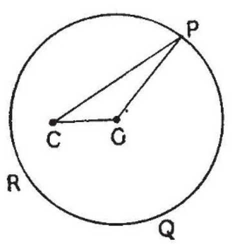


FIG. 3.

and integrating this with respect to θ from π to 0 we have for the relative mean values of dr, caused respectively by alterations of dR and $-d\rho$ in the values of R and ρ,

$$\left\{\int_0^{\pi}\frac{dr}{dR}\bigg/\int_0^{\pi}\frac{dr}{d\rho}\right\}d\theta = \frac{\rho}{R}.$$

If the two groups are equally good, the mean value of dR must be equal to minus the mean value of $d\rho$.

This leads to a simple relation between R and ρ, viz. $R^2 + \rho^2$ = constant. Thus any group of shots for which the sum of the squares of the mean distance of the group from the centre and its radius of gyration is constant is equally good.

This may be stated more concisely by saying that when the object is to hit the centre of the target, the merit of any series of shots is inversely proportional to its moment of inertia of the group about the centre of the target.

If for convenience it is decided to make the score 100 when the moment is unity, the worth of any given series will be represented by

$$\frac{100}{R^2 + \rho^2}.$$

I give below an actual target with the results analysed in the way described.

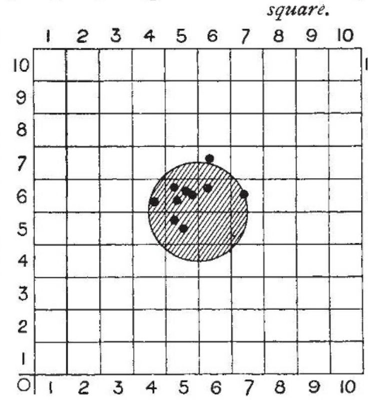
If a slide rule is used, the arithmetic takes about five minutes.

I do not for a moment suppose that such an analysis would be practicable at ordinary rifle matches, but it does seem possible that coordinate targets might become popular, and some simple way devised of using the more precise information they would afford.

As far as finding the moment of inertia of the group about the centre of the target is concerned, this might be done more simply by the use of polar coordinates, only the mean square of the distance from the centre being wanted for that purpose; but the Cartesian coordinates are more convenient when the closeness of the grouping has to be considered.

EXAMPLE.

Coordinate Target. Ten shots at 100 *yards. Target* 10 *inches square.*



Shot	Score X	Score Y
1	5	6
2	4	6
3	5	5
4	5	6
5	6	7
6	7	6
7	5	6
8	6	6
9	5	6
10	5	5

FIG. 4.

To analyse the score it is convenient to arrange the results in the following form; p and q being the number of shots on each vertical and horizontal row.

x	p	px	y	q	qy	px^2	qy^2
1	—	—	1	—	—	—	—
2	—	—	2	—	—	—	—
3	—	—	3	—	—	—	—
4	1	4	4	—	—	16	—
5	6	30	5	2	10	150	50
6	2	12	6	7	42	72	252
7	1	7	7	1	7	49	49
8	—	—	8	—	—	—	—
9	—	—	9	—	—	—	—
10	—	—	10	—	—	—	—
		$\frac{\Sigma x}{n}$ 5·3			$\frac{\Sigma y}{n}$ 5·9	$\frac{\Sigma x^2}{n}$ 28·7	$\frac{\Sigma y^2}{n}$ 35·1
		$\frac{\Sigma x}{n} - h =$ 5·5			$\frac{\Sigma y}{n} - k =$ 5·5	$\left(\frac{\Sigma x}{n}\right)^2$ 28·1	$\left(\frac{\Sigma y}{n}\right)^2$ 34·7
		−·2			·4	·6	·4
							.6

$R^2 = (\cdot 2)^2 + (\cdot 4)^2 = \cdot 2$ $\qquad \rho^2 = 1{\cdot}0$

This counts $\frac{100}{1 + 2{\cdot}2}$ or 84.

A. MALLOCK.