

triously and ably worked out, and, on the whole, well written. At the same time, it should be pointed out that such a work was especially in need of a good and exhaustive index, and that it is a pity the author did not compile one himself.

#### CALCULATION BY ABACUS.

*Traité de Nomographie.* Par Maurice d'Ocagne. Pp. xiv + 480. (Paris: Gauthier-Villars, 1899.)

THIS is a book which ought to make even the ordinary reader appreciate the perennial freshness of mathematics. The method of "Nomography" (X3 of the international catalogue), recent as it is in its more important developments, is based upon a very simple idea which has long been familiar—that of the indexed scale. The ever-recurring problem of applied mathematics is to calculate an unknown numerical quantity from its relation to other quantities that are known. The simplest case is when two quantities  $x, y$  are connected by a relation  $f(x, y) = 0$  or  $y = \phi(x)$ . For practical purposes it is convenient to have a permanent record of a large number of corresponding values of  $x$  and  $y$  so that for any given value of  $x$  the approximate value of  $y$  may be at once found or obtained by simple interpolation. Three methods are available: the first is that of a numerical table, such as a table of logarithms; the second that of the graph, for instance the curve  $f(x, y) = 0$  or  $y = \phi(x)$  referred to rectangular coordinates; the third is that of the indexed scale, that is to say a straight line or curve at different points of which the corresponding values of  $x$  and  $y$  are shown in figures. A familiar example is given by a thermometer with Centigrade and Fahrenheit readings, or by a measuring tape with centimetres marked along one edge and inches along the other.

In this very simple case the advantage of the indexed scale is not very obvious; even here, however, the method combines much of the vividness of the graph with a considerable saving of space. It is when three or more variables are connected by a relation that the great value of the scale method becomes apparent. Suppose, for instance, we have a relation

$$F\{\phi(x), \chi(y), \psi(z), \omega(t)\} = 0$$

where  $x, y, z, t$  are the variables and  $F, \phi, \chi, \psi, \omega$  are known functions. The essence of the nomographic function consists in first plotting off in a suitable way indexed scales of  $\phi(x), \chi(y), \psi(z), \omega(t)$ , and then employing a linkage or similar mechanism to associate four corresponding values,  $x', y', z', t'$ . In the case of two variables  $x, y$  the "linkage" consists merely in the juxtaposition of the scales; when a proportion sum is done with a slide-rule, the scales are moved relatively to each other; in most of M. d'Ocagne's illustrations, involving several variables, the scales are either superposed in a two-dimensional grating or a movable linkage is used consisting of a transparent sheet with lines of reference ruled upon it, or a combination of both devices is employed.

Of course a method so elastic leaves ample room for ingenuity in constructing an "abacus," as M. d'Ocagne calls it, suited to any particular problem. The author

gives an abundant variety of illustrations, many of great practical importance to the physicist and engineer: it is by studying these, and actually taking readings for himself, that the reader will succeed in appreciating the value of the method. For of this, as of other graphical methods, it may be said that merely reading it up, or understanding its principles in a general way, is of little use as compared with a thorough working knowledge of its application.

At the same time, M. d'Ocagne has done really good service in devoting his final chapter to the general theory. This has, in its way, the same kind of special value as Reuleaux's "Kinematics of Machinery" in relation to the ordinary treatises on mechanism. For in this chapter we have a clear conspectus of the general principles which underlie the construction of *any* abacus; and, what is still more remarkable, all possible varieties of abacus are classified into perfectly definite types which can be expressed by a simple abstract notation. Oddly enough, the enumeration of the different types leads to a difficult problem in the partition of numbers, happily solved by Major MacMahon.

It is not impossible that the human race may ultimately set off against the ravages of warfare the indirect stimulus which it has given to mathematics; nomography, at any rate, has been developed in great measure to meet the demands of civil and military engineering. M. d'Ocagne's numerous bibliographical notes will enable his readers to follow in detail, if they wish, the history of the subject. Pure and applied mathematicians alike will be grateful to him for a work so full of novelty and interest; while its subject-matter, as well as its clearness and simplicity, ought to make it eminently acceptable to the engineer.

G. B. M.

#### OUR BOOK SHELF.

*Die Spiele der Menschen.* By Karl Groos. Pp. vi + 538. (Jena: G. Fischer, 1899.)

PROF. GROOS will add by the present volume to the reputation he has already earned by his well-known work on the "Games of Animals." A really comprehensive account, at once sympathetic and intelligent, of the games of both children and adults has long been a desideratum with the psychologist as well as with the anthropologist, and Prof. Groos's new work goes very far indeed towards permanently supplying the want. As is only right and proper, by far the larger part of the book is given up to an exhaustive description of the facts as far as they are known; the "Theory of Play" enunciated in the second part of the treatise can thus be judged by the reader upon a sufficiently wide basis of empirical information. The range and the accuracy of Prof. Groos's knowledge are alike surprising; not only is he a mine of information about the amusements of his own country, but he appears, for instance, as much at home in the English nursery and playground as though he had been brought up amongst us. Almost the only signs of imperfect knowledge of English games to be detected in the whole book are the author's ascription of "Hare and Hounds," in its familiar form, exclusively to America, and his apparent ignorance of the continued vitality of "Hunt the Slipper." As a psychologist Prof. Groos is distinguished by a singular subtlety of discrimination; his account, for instance, of the various elements which enter into the gambler's enjoyment of high play, or, again