LETTERS TO THE EDITOR.

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Velocity of Propagation of Electrostatic Force.

As we may have to wait some time for the experimental solution of Lord Kelvin's very instructive and suggestive problem concerning two pairs of spheres charged with electricity (see NATURE of February 6, p. 316), it may be interesting to see what the solution would be from the standpoint of existing electrical theories.

In applying Maxwell's theory to the problem, it will be convenient to suppose the dimensions of both pairs of spheres very small in comparison with the unit of length, and the distance between the two pairs very great in comparison with the same unit. These conditions, which greatly simplify the equations which represent the phenomena, will hardly be regarded as affecting the essential nature of the question proposed.

Let us first consider what would happen on the discharge of (A, B), if the system (c, d) were absent.

Let m_0 be the initial value of the *moment* of the charge of the system (A, B), (this term being used in a sense analogous to that in which we speak of the *moment* of a magnet), and m the value of the moment at any instant. If we set

$$m = \mathrm{F}(t), \quad \ldots\ldots\ldots \quad (1)$$

and suppose the discharge to commence when $t = 0$, and to be completed when $t = h$, we shall have

$$\mathrm{F}(t) = m_0 \qquad \text{when} \qquad t < 0, \quad \ldots \quad (2)$$

and

$$\mathrm{F}(t) = 0 \qquad \text{when} \qquad t > h, \quad \ldots \quad (3)$$

Let us set the origin of coordinates at the centre of the system (A, B), and the axis of χ in the direction of the centre of the positively charged sphere. A unit vector in this direction we shall call i, and the vector from the origin to the point considered ρ. At any point outside of a sphere of unit radius about the origin, the electrical displacement $(\mathfrak{D})$ is given by the vector equation

$$4\pi\mathfrak{D} = [3r^{-5}\mathrm{F}(t - cr) + 3cr^{-4}\mathrm{F}'(t - cr) + c^2r^{-3}\mathrm{F}''(t - cr)]\chi\rho \\ -[r^{-3}\mathrm{F}(t - cr) + cr^{-2}\mathrm{F}'(t - cr) + c^2r^{-1}\mathrm{F}''(t - cr)]i, \quad . \ (4)$$

where F denotes the function determined by equation (1), F′ and F″ its derivatives, and c the ratio of the electrostatic and electromagnetic units of electricity, or the reciprocal of the velocity of light. For this satisfies the general equation

$$-\nabla^2\mathfrak{D} = c^2d^2\mathfrak{D}/dt^2, \quad \ldots\ldots\ldots \quad (5)$$

as well as the so-called "equation of continuity," and also satisfies the special conditions that when $t < 0$

$$4\pi\mathfrak{D} = m_0(3r^{-5}\chi\rho - r^{-3}i)$$

outside of the unit sphere, and that at any time at the surface of this sphere

$$4\pi\mathfrak{D} = m(3\chi\rho - i),$$

if we consider the terms containing the factor c as negligible, when not compensated by large values of r. That equation (4) satisfies the general conditions is easily verified, if we set

$$u = r^{-1}\,\mathrm{F}(t - cr), \quad \ldots\ldots\ldots \quad (6)$$

and observe that

$$-\nabla^2 u = c^2d^2u/dt^2, \quad \ldots\ldots\ldots \quad (7)$$

and that the three components of $\mathfrak{D}$ are given by the equations

$$\left.\begin{aligned} 4\pi f &= -d^2u/dy^2 - d^2u/dz^2 \ \ldots \\ 4\pi g &= d^2u/dxdy \ \ldots\ldots\ldots \\ 4\pi h &= d^2u/dxdz \ \ldots\ldots\ldots \end{aligned}\right\} (8)$$

Equation (4) shows that the changes of the electrical displacement are represented by three systems of spherical waves, of forms determined by the rapidity of the discharge of the system (A, B), which expand with the velocity of light with amplitudes diminishing as r^{-3}, r^{-2}, and r^{-1}, respectively. Outside of these waves, the electrical displacement is unchanged, inside of them it is zero.

If we write (with Maxwell) $-d\mathfrak{A}/dt$ for the force of electrodynamic induction at any point, and suppose its rectangular components calculated from those of $-d^2\mathfrak{D}/dt^2$ by the formula used in calculating the potential of a mass from its density, we shall have by Poisson's theorem

$$\nabla^2(d\mathfrak{A}/dt) = 4\pi d^2\mathfrak{D}/dt^2,$$

or by (5),

$$\nabla^2(d\mathfrak{A}/dt) = -4\pi c^{-2}\nabla^2\mathfrak{D},$$

whence

$$d\mathfrak{A}/dt = -4\pi c^{-2}\mathfrak{D} \quad \ldots\ldots\ldots \quad (9)$$

From this, with (4), and the general equation

$$d\mathfrak{A}/dt + 4\pi c^{-2}\mathfrak{D} + \nabla\mathrm{V} = 0,$$

we see that during the discharge of the system (A, B) the electrostatic force $-\nabla\mathrm{V}$ vanishes throughout all space, while its place is taken by a precisely equal electrodynamic force $-d\mathfrak{A}/dt$.

This electrodynamic force remains unchanged at every point until the passage of the waves, after which the electrostatic force, the electrodynamic force, and the displacement, have the permanent value zero.

If we write *Curl* for the differentiating vector operator which Maxwell calls by that name, equations (8) may be put in the form

$$4\pi\mathfrak{D} = \text{Curl Curl } (iu),$$

whence

$$d\mathfrak{D}/dt = (4\pi)^{-1} \text{ Curl Curl } (idu/dt).$$

From $d\mathfrak{D}/dt$ we may calculate the magnetic induction $\mathfrak{B}$ by an operation which is the inverse of $(4\pi)^{-1}$ Curl. We have therefore

$$\mathfrak{B} = \text{Curl } (idu/dt),$$

or

$$\mathfrak{B} = [r^{-3}\mathrm{F}'(t - cr) + cr^{-2}\mathrm{F}''(t - cr)](yk - zj).$$

The magnetic induction is therefore zero except in the waves.

Equations (4) and (9) give the value of $d\mathfrak{A}/dt$ as function of (t and r). By integration, we may find the value of $\mathfrak{A}$, Maxwell's "vector potential." This will be of the form of the second member of (4) multipled by $-c^{-2}$, if we should give each F one accent less, and for an unaccented F should write $\mathrm{F}_{,}$, to denote the primitive of F which vanishes for the argument ∞.

That which seems most worthy of notice is that although simultaneously with the discharge of the system (A, B) the values of what we call the electric potential, the electrodynamic force of induction, and the "vector potential," are changed throughout all space, this does not appear connected with any physical change outside of the waves, which advance with the velocity of light.

If we now suppose that there is a second pair of charged spheres (c, d), as in the original problem, the discharge of this pair will evidently occur when the relaxation of electrical displacement reaches it. The time between the discharges is, therefore, by Maxwell's theory, the time required for light to pass from one pair to the other.

It may also be interesting to observe that in the axis of χ, on both sides of the origin, $\chi\rho = r^2 i$, and equation (4) reduces to

$$4\pi\mathfrak{D} = [2r^{-3}\mathrm{F}(t - cr) + 2cr^{-2}\mathrm{F}'(t - cr)]i.$$

Here, therefore, the oscillations are normal to the wave-surfaces. This might seem to imply that plane waves of normal oscillations may be propagated, since we are accustomed to regard a part of an infinite sphere as equivalent to a part of an infinite plane. Of course, such a result would be contrary to Maxwell's theory. The paradox is explained if we consider that the parts of the wave-motion, expressed by F and F′, diminish more rapidly than those expressed by F″, so that it is unsafe to take the displacements in the axis of χ as approximately representing those at a moderate distance from it. In fact, if we consider the displacements not merely in the axis of χ, but within a cylinder about that axis, and follow the waves to an infinite distance from the origin, we find no approximation to what is usually meant by plane waves with normal oscillations.

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New Haven, Conn., March 12.

An Unusual Solar-Halo.

On March 17, at Göttingen, a curious solar halo was observed by a friend and myself towards the time of sunset. The weather that day had been beautifully fine, but towards 5h. p.m. (Mean European Time) thin light clouds began to form, which covered the heavens with a thin white raiment. When the sun was