

the curves for which the real part u of w has a constant value are algebraical, then w is either an algebraical function of z , or μw , where μ or μz is some real number, is the logarithm of an algebraical function, or else μw is an elliptical integral of the first kind, with real modulus whose superior limit is an algebraical function of z .

All these investigations require a deep study of the very foundations of analysis. These cannot help to reveal a number of inaccuracies and gaps in the ordinary theories as contained in text-books. Accordingly we find several papers in the collection which contain such corrections. Thus there is one paper in which a complete system of independent conditions is given which underlie the proposition that $d^2u/dx^2dy^2 = d^2u/dy^2dx^2$.

In another paper the definition of the area of a curved surface in the first edition of Serret's "Calcul Différentiel et Intégral" is shown to be wrong. In another long paper it is proved that of all solids of given volume the sphere has the smallest surface. All previous proofs depend on the supposition that *one solid exists* which has a minimum surface; of this the present proof is independent.

The first volume contains papers relating to surfaces of minimum area. The original problem is one of the calculus of variation, viz. a given closed curve in space being given, it is required to determine that surface bounded by it which has the least area. One of the chief properties of such surfaces is that the principal radii of curvature at each point are equal but opposite, or the mean curvature is everywhere = 0. Hence all surfaces which have this property are called "surfaces of minimum area," though it is not any longer true that surfaces of this kind have the original property for every part cut out by any curve drawn on them, just as the arc of a great circle on a sphere ceases to be the shortest line between its ends as soon as the arc becomes greater than a semicircle. The question to decide whether this is the case for a given closed curve on the surface is considered in the paper "Ueber ein die Flächen kleinsten Flächeninhalts betreffendes Problem der Variationsrechnung." It is, of course, a problem about the second variation.

There is an interesting connection between the surfaces of minimum area and the conform representation of one surface upon another, viz. every such representation of the whole surface of a regular polyhedron on a sphere gives rise to a surface of minimum area, which in this case contains an infinite number of straight lines.

In the first paper the surface is considered which is thus obtained from the cube. This paper, when communicated to the Berlin Academy, was illustrated by models. In the present reprint, nicely executed shaded figures of these are added. The first gives the surface corresponding to one face of the cube. It is bounded by four edges of a regular tetrahedron. In the second we have the surface corresponding to all six faces of the cube. It forms one continuous sheet. If this surface be still further continued, a surface is obtained which extends throughout the whole space. Of this the third plate gives a part. Analytically the problem depends on elliptical functions.

These investigations are continued in the second paper (and several others), which obtained a prize of the Berlin Academy. The prize problem required the complete solution, by aid of elliptical or Abelian functions, of some important problem taken from almost any part of pure or applied mathematics. Herr Schwarz treats of the surface of least area bounded by any skew quadrilateral.

In "Fortgesetzte Untersuchungen über specielle Minimalflächen," a new problem is proposed, viz. there is given a closed chain consisting of straight lines and planes, it is required to find a surface of minimum area bounded by the lines and perpendicular to the planes.

Of the other papers we mention the "Miscellen aus

dem Gebiete der Minimalflächen." It contains a highly interesting review of the whole subject, including Plateau's investigations, and is full of suggestions.

The volumes, which are dedicated to Weierstrass, are well printed on octavo pages sufficiently large to give room for the formulæ required, and not so large as to be unwieldy, as is the case with a recently published "Collection." But there is one point in which the edition might have been improved, trifling as far as editing and printing are concerned, but of great benefit to the reader. It is very desirable that in all editions of collected papers the examples set by Sir William Thomson and Prof. Cayley should be followed, of placing the date of the first publication of each paper both in the table of contents and at the head of each paper.

O. HENRICI.

GEOGRAPHICAL EXPEDITIONS.

M. GROMBCHESKY, now at St. Petersburg, has given the Russian Geographical Society a most interesting account of his last expedition. It is known that the Expedition left Marghelan in June 1889, and that having found the Alai Mountains deeply clothed in snow, they went to Kala-i-khumb through Karategin and Vakhia. They found that the khanate of Shugnan was at war with the Afghans, and as the latter refused to let the Expedition go further, M. Grombchevsky returned to Vakhia, after having crossed the Sytarghi Pass, which has on its western slope a great glacier, six miles long. In August, after having made a long circuitous journey over the Pamir (the well-known Pamir robber, Sahir-Nazar, being the guide of the Expedition), they reached the frontier of the Pamir khanates now occupied by the Afghans, and waited there for the Ameer's permission for further advance. A refusal was received in October, when the temperature already was from 20° to 24° C. below zero, and the Expedition could find no fuel of any kind. So they crossed the Mus-tagh ridge (yaks being used for the transport of provisions), and reached the valley of the Raskem River, where they met with Mr. Young-husband. During their fifty-five days' stay on the banks of the Raskem, they explored the passes of Shimshal, Mustagh, and Balti-davan, leading to Kashmir, as well as the passes across the Raskem ridge leading to Kashgaria. In November, M. Grombchevsky was at the Kashmir fort Shahidulla-kodja; but the fort was abandoned, and, the Expedition having no provisions, they asked permission to enter Kashmir and to winter there. But Colonel Nisbet refused admission to Kashmir, so that the Expedition had nothing to do, M. Grombchevsky says, but to move eastward, across the desert plateaus of Tibet, in order to reach some inhabited spot. Moving up the Kara-kash, the Expedition ascended the Tibet plateau. The thermometer fell as low as - 33° to - 35° C., all water was frozen, and two-thirds of the horses died; so that all natural history collections were abandoned, and, notwithstanding a frightful snowstorm, the Expedition re-crossed the mountains and went to Kashgaria. The first settlements were reached in February. Next month M. Grombchevsky went to Khotan, and thence to Niya, where he met with the commander of the Tibet Expedition, M. Pyevtsoff. At the end of March, he visited the Sourgak gold-mines in the south of Niya—where he found 3000 men busy in gold-washing—and Polu, whence he again ascended the Tibet plateau, and after some explorations he returned to Kashgaria again. In the autumn he visited the middle course of the Raskem River, making acquaintance with interesting tribes of mountaineers, and thence returned to Russia. The geographical results of the expedition seem to be very important. Surveys were made over a length of 5000 miles, and latitudes and longitudes were determined at 73 different spots; heights were measured throughout the journey, and photographic

views taken; and rich geological, botanical, and entomological collections were secured.

On January 31, in the great amphitheatre of the Sorbonne, Paris, the French Geographical Society held a special meeting for the reception of M. Gabriel Bonvalot and Prince Henry of Orleans, whose travels in the heart of Central Asia have won for them an honourable place in the ranks of modern explorers. The chair was taken by M. de Quatrefages, who warmly congratulated the explorers on their achievements, and announced that the Society had conferred on the Expedition its large gold medal, the highest reward at its disposal.

M. Bonvalot, the chief of the Expedition, gave a full and interesting account of the journey. He left Paris with Prince Henry on July 6, 1889, and arrived on September 1 at the Russo-Chinese frontier, near which their caravan was organized. At Kuldja they met Father Dedékens, a Belgian missionary, who, to their great satisfaction, consented to accompany them, and rendered them important services. Having crossed the mountains of Tian-Shan, they arrived at Kurla, in Chinese Turkestan, where M. Bonvalot engaged fresh camels. At Lake Lob-Nor they reorganized their caravan, and laid in stores for six months. They then crossed the chains of Altyn Tagh, the Tshimen Tagh, and the Columbo Mountains, travelling sometimes at heights of more than 4000 metres. The region was wild and desolate, and the cold intense; and M. Bonvalot found it necessary to limit the Expedition to fourteen men, forty camels, and eighteen horses, the rest being sent back. Having followed for some time the traces of a caravan in the direction of Lhasa, he decided to keep to the same route as far as it could be made out; and in his address at the Sorbonne he gave a vivid description of the difficulties the party encountered in trying to discover the way the caravan had taken. On December 31, at a height of more than 5000 metres, a terrible storm caused them to lose sight of the marks by which they had been guided; whereupon they journeyed along the 90th degree of longitude. They found great chains of mountains, vast lakes, extinct volcanoes, geysers, and a Pass at an altitude of 6000 metres. Below 5000 metres they met with herds of wild yaks, antelopes, and other animals. Birds had wholly disappeared, and there was no vegetation. The only water they could obtain was melted ice, and cooking was impossible. Two men died, and the animals perished one after another. At last the traces of the route were discovered, and the Expedition arrived at Lake Ten'gri-Nor, where they met certain Tibetan authorities, who were accompanied by numerous horsemen. They had great difficulty in proving that they were Frenchmen, but after forty-five days of negotiation, at Dam, near Lhasa, the Tibetans provided them with the means of continuing their journey, as they had lost all their own means of transport.

The travellers followed what is called "the little route" from Tibet to China—a route still unexplored. They crossed the territory of independent tribes, who, in accordance with the wishes of the Llama, furnished them with yaks and horses. They were now in a region of valleys, and of wooded grounds well supplied with game and with large wild animals. In the course of three days they saw twenty-two bears. Some of the valleys are cultivated and occupied by villages. The Expedition followed the upper courses of the Salouen and the Mékong, and that of the Yang-tse-kiang, the sources of which they thought they recognized on the southern side of a colossal chain of mountains which they called "Monts Dupleix."

At Batang, which they reached on June 7, 1890, they met with Chinamen. They rested for a month at Tsiensien-Lou, on the Chinese frontier, where they received a

cordial welcome from French missionaries; and on July 29, they started for Tonkin, arriving at Yunnan on September 5, where they found a letter from Europe, dated September 5, 1889. Reaching Manghao, on the Red River, they hired Chinese junks, and entered Tonkin at Lao-Kai. Soon afterwards they were at Hanoi. Altogether, they had traversed 2500 kilometres on an unknown route.

Among the more important of the geographical results of the journey is the discovery of volcanic regions. On December 22, 1889, they observed on the plateau they were crossing a *coulée* of lava; and, looking towards the horizon, they saw in the west an isolated volcano, to which they gave the name of Mount Reclus, in honour of the well-known geographer. Further on, they came to other volcanoes, near which they saw great blocks of lava, which at a distance they took for yaks. One small chain reminded them of the mountains of Auvergne.

In the great chain of Dupleix they found fossils (bivalves), belonging to Tertiary strata, at a height of 5800 metres. In the same region they discovered various minerals, especially iron and lead. At the foot of the Dupleix chain, among rocks, they met with grey monkeys, with rather long hair and short tails. These creatures appeared to be isolated, as they had not been seen before, and were not seen afterwards.

At the meeting of the Royal Geographical Society on Monday, Mr. E. G. Ravenstein gave some account of the British East Africa Company's Expedition, under Mr. F. J. Jackson, from Mombassa to Uganda. The route up to Machako's, about 250 miles north-west of Mombassa, is already pretty well known from the narratives of Mr. Joseph Thomson and others. The portion between Machako's and Uganda had also been traversed to some extent by Mr. Thomson, as well as by Count Teleki and the late Dr. Fischer. Captain Lugard found that the plateau, which rises to about 6000 feet at Machako's, is much broken up by ravines, while there are numerous waterless stretches, where, however, water can generally be found by digging. There are numerous valleys and glades, with abundant vegetation; many patches of forest, mostly of soft-wood trees, and even several perennial streams. Iron and copper are abundant in some places, and indications of gold were found by Captain Lugard. From Machako's, Mr. Jackson's caravan had to make its way up the steep face of the Kinangop escarpment, 9000 feet in altitude, below which, in the valley between that and the equally steep and high Mau escarpment lay lake Naiwasha, and several other lakes, all without outlets, and yet all fresh. A descent of some 3000 feet has to be made to the lakes. These two escarpments, which may be said to extend more or less continuously from Abyssinia to Ugogo, are, Mr. Ravenstein pointed out, two of the most remarkable physical phenomena on any continent. The plateau between Machako's and Lake Victoria Nyanza is even more broken up by deep ravines than that between Machako's and the coast, so that travelling becomes of the most trying character. While the country here is to a large extent of a steppe character, still there are some districts of the highest fertility. In some cases the forest has been cleared away, and the country cultivated by the natives, some tribes being great cattle-rearers. Many of the gorges are still densely clad with forests, and beyond the Mau escarpment is a perfect network of rivers. Game was plentiful and buffaloes were seen in large herds. The north-east corner of Lake Victoria Nyanza has been laid down more accurately than on existing maps, and the contour given to it by Mr. Stanley is in all essential respects confirmed. Usogo, where the Expedition received a cordial welcome, is evidently one of the richest countries in Africa; a marked contrast to Uganda, which, owing to the strife which has prevailed since the death of Mtesa, has been converted into a wilderness

Before entering Usogo, Mr. Jackson made a detour to the north-east of Mount Elgon, but did not succeed in reaching Lake Rudolph, visited by Count Teleki. The country in this direction is of a barren steppe character, sparsely covered with bush, and with a few heights rising above the general level. On his way back, Mr. Jackson and his caravan travelled right across the summit of Mount Elgon, one of the most remarkable mountains in Africa. It is an extinct volcano, the crater of which is eight miles in diameter, its appearance reminding one of the great craters seen in lunar photographs. This mountain is over 14,000 feet high, and, taken in combination with Kilimanjaro, Kenia, and Ruwenzori, seems to indicate that at one period this must have been a region of intense volcanic activity. High up on the face of this mountain Mr. Jackson came upon the caves of which Mr. Thomson told us. These he found to be entirely natural, and not the work of man. One is so large that on its floor has been built a village of huts; for the caves are inhabited by natives who have been compelled to take refuge here from their enemies in the plains. Mr. Jackson's natural history collections are very extensive; very many new species of birds and insects have been sent home. Mr. Bowdler Sharpe stated that these collections have revolutionized existing notions as to the zoological geography of Africa. In the Mount Elgon region types are found similar to those of Abyssinia on the one hand and the Cape on the other; and Mr. Sharpe stated that the region most resembling that of Elgon is that of the Cameroons Mountains in West Africa; but this is based mainly on the ornithology of the two regions, the entomology leading to somewhat different conclusions. On the whole, the geographical and natural history results of the expedition are of high importance, and credit is due to the British East Africa Company for encouraging work of this kind.

NOTES.

PROF. HELMHOLTZ, as we have already stated, will celebrate his seventieth birthday on August 31. In honour of the anniversary, a marble bust of Prof. Helmholtz will be prepared; and it is proposed that there shall be a Helmholtz Medal, to be bestowed on the most eminent German and foreign physicists. An international committee has been formed for the purpose of carrying out these schemes.

AT its last meeting, January 28, the Russian Geographical Society awarded its two great gold Constantine Medals to Prof. A. Potebnya for his numerous ethnographical and philological researches, and to Prof. Th. Sloudsky for his geodetical work. The Count Lütke's medal was awarded to S. D. Rylke, also for geodetical work, and especially for the mathematical discussion of the results of the recent exact levellings. Gold medals were awarded to P. Rovinsky for his geographical and ethnographical description of Montenegro; to N. Filipoff for his work on the changes of level of the Caspian Sea; to V. Obrutcheff for a work on the Transcaspien region; and to V. Priklonsky for his manuscript, "Three Years in the Yakutsk Region." A number of silver medals were awarded to several persons for many years' meteorological observations, and various ethnographical works of minor importance.

THE ninth German Geographentag, which will meet in Vienna on April 1, 2, and 3, will deal chiefly with the present state of our geographical knowledge of the Balkan peninsula, and with the investigation of inland seas. A geographical exhibition will be held in connection with the meeting.

A ROYAL Commission has been appointed to inquire into the effect of coal-dust in originating or extending explosions in

coal-mines. Mr. Chamberlain is to be the Chairman, and his brother Commissioners are Lord Rayleigh, Sir William Lewis, Prof. Dixon, Mr. Emerson Bainbridge, and Mr. Fenwick, M.P. A small committee of experts has already investigated the subject.

ON Friday last, in the House of Commons, in answer to Sir H. Roscoe, Mr. Plunket said it had been decided to proceed at once with the completion of the buildings in connection with the Science and Art Department on the east side of Exhibition Road. They would ultimately be devoted wholly to art collections, although for some years it was probable that some portions of them might be temporarily available for science collections. The buildings would, however, take several years to complete, and he was in communication with the Science and Art Department as to the best means of providing for the science requirements, and he hoped soon to be able to submit a proposal to the Treasury. As the buildings on the east side of Exhibition Road would cost some £300,000 or £400,000, it was obvious that any further immediate demands on the Chancellor of the Exchequer must be confined within as narrow limits as possible.

SOME time ago Sir Joseph Fayrer announced his intention of retiring from the presidency of the Sanitary Assurance Association. At the tenth annual meeting of the Association on Monday, Mr. Rutherford referred to the great services Sir Joseph had rendered to the cause of sanitary improvement, and proposed a resolution expressing warm appreciation of the valuable services rendered by him during the ten years in which he had held the office of President, and assuring him of "their high admiration for the zeal and energy manifested in his disinterested and gratuitous discharge of those services (amidst numerous other public and private duties) to the undoubted promotion of the general health and public good." This was seconded by Surgeon-General Cornish, and warmly supported by Prof. Smith, and adopted. Sir Joseph Fayrer, in acknowledging the resolution, said he should continue to take a lively interest in the Association and its work, and would retain his seat on the Council. Surgeon-General Cornish, late Sanitary Commissioner with the Madras Government, was elected President; and Sir Joseph Fayrer and Prof. Roger Smith were elected Vice-Presidents.

A CAPITAL paper on decimal coinage, weights, and measures was delivered before the Society of Arts on February 4 by Mr. J. Emerson Dowson, and is printed in the current number of the Society's Journal. He urged that there is pressing need for a thorough investigation of the whole subject by a Royal Commission; and, as evidence of the opinion of important bodies of commercial men on the subject, he mentioned that all the seventy-two Chambers of Commerce of the Association of the United Kingdom have repeatedly pronounced themselves in favour of the decimal system, and that the four large Chambers which are not members of the Association (Edinburgh, Glasgow, Liverpool, and Manchester) have taken the same ground. Sir Henry Roscoe, who presided, remarked that he himself had largely benefited by the use of the metric system, which was employed of necessity by men of science in all countries. He had also seen the simplicity and ease with which arithmetic was taught in foreign schools, and could bear testimony to the way in which the old German system of weights and measures was entirely swept away in a few months, when the new system became the law of the land, and how readily it was adopted by the people. Mr. Goschen told Mr. Leng, in answer to the memorial sent from the Dundee Chamber of Commerce, that he was well aware of the strong case which the advocates of a decimal system had made out, but the difficulties were very great, and he could not undertake to bring in a Bill. They did