

increases, and of this electric glow many instances could be cited, both in Nature and in the laboratory.

Consider, in the first place, the glow surrounding a point from which an electric discharge is taking place. By means of the electrical repulsion, the density of the air immediately surrounding the point will be so far diminished that a single air-particle will be able to traverse a sensible distance with a very great velocity, and therefore give rise to the glow. Here it is not a question of particles becoming electrically excited by radiation from the point, but of those which are electrified by actual contact with it. As soon as they have lost some of their electrical energy they will again become sensitive to electrical radiation. There must therefore be a dark space immediately surrounding the point, and outside this an electric glow, which explains a well-known phenomenon always observed in the rarefied atmosphere of a Geissler tube. The stratification can also be explained very simply, for the glow causes a diminution in velocity, for when the electrical waves from the positive electrode give rise to luminous instead of electrical vibrations in the particles of gas, the repulsion will be diminished, and therefore the velocity will gradually become less than that of light, when the particle will again become sensitive to the electrical radiation. The velocity will therefore again increase until the glow appears again, thus giving rise to a stratified appearance. The velocity in the glowing layers will naturally be greatest in the neighbourhood of the positive electrode, and here, therefore, light will be given off of all the colours corresponding to the critical periods of the gas contained in the tube, which is in accordance with observation. According to the author's theory, the electrical excitation takes place by the transference of ponderable gas molecules from the positive to the negative electrode. After they have parted with their electrical energy to the latter, they will return in an unelectric condition to the positive electrode to which they will be attracted, and at the same time repelled from the negative electrode. There will be no dark space surrounding the negative electrode, because the particles leaving it will have little or no electrification. The velocity of the returning molecules will increase as they approach the positive electrode, so that there can be no further transformation of electrical into luminous energy. In very high vacua the velocity of the returning particles may become great enough for electrical energy to be excited in them by the red glow of the positive pole, by which their velocity will be still further increased. The velocity of the returning particles will in this case ultimately become so much greater than that of the luminous molecules moving away from the positive electrode as to cause a sensible increase in the density of the gas surrounding it. The result of this will be to prevent the formation of the positive glow, and the whole tube will become filled by the negative glow. The density in the neighbourhood of the negative electrode will therefore be diminished, and the returning molecules will leave it with still greater velocity. If both electrodes are at one end of the tube, the molecules returning towards the positive electrode will be deflected by the layer of dense gas surrounding it, against the sides of the tube, giving rise to fluorescent phenomena, as explained in § 11 (September 6, p. 461). If the complicated phenomena which have recently been observed in Geissler tubes by Crookes and Hittorf can be thus simply explained, it will afford an important confirmation of the author's theory.

These considerations may be applied to the explanation of many cosmical phenomena, such as the aurora and the light of comets. It is quite possible that the particles of a comet's tail moving with great velocity towards the sun may become electrified by means of the sun's light.

The formulæ previously obtained are applicable to the determination of the motion of an electrified particle, in the case in which no proper luminous vibrations are given off from the origin, or where these may be neglected, for the equations

(29) to (33) give in this case for $\frac{dr}{dt} = c$, $r = r_0$, $\mathfrak{R} = \mathfrak{R}_0$, and consequently—

$$\frac{\mu}{2} \left(1 - \frac{\rho}{r_0} \right) c^2 = M \left(\frac{1}{r} - \frac{1}{r_0} \right) + \mathfrak{R}_0.$$

Also—

$$\frac{\mu}{2} \left[\left(\frac{dr}{dt} \right)^2 - c^2 \right] = - \frac{2M}{r} + \frac{M}{r_0} + \frac{Mc}{r \frac{dr}{dt}} + \mathfrak{R} - \mathfrak{R}_0.$$

And dr/dt can hence only become infinite when the positive quantity \mathfrak{R} becomes infinite, or $r = 0$. von Helmholtz's objections, therefore, do not apply to this equation.

§ 15.—*Electrical Excitation.*

The foregoing theory easily explains the different methods of electrical excitation.

(1) The friction of two bodies sets their molecules into vibration, which appears in the form of heat. The resulting impacts of neighbouring molecules will most readily excite internal vibrations of the critical periods, for which they are specially sensitive. If the molecules are exceptionally sensitive to vibrations of very short periods, they will be easily electrified, the process being exactly analogous to the production of luminous vibrations by heating gases, as described in § 4 (August 23, p. 407). Electro-positive bodies will be those which are most sensitive, and these will, according to the theory, attract other less electrified bodies. In the ordinary frictional electrical machine the glass will therefore be more strongly excited than the rubber. The explanation of the collecting action of points on the prime conductor is given by the consideration that at a point the molecules are more fully exposed to the electrical radiation from the glass plate, and being electrically excited by this radiation communicate their electrification to the prime conductor by conduction, as explained in § 13.

(2) Electrification by the action of heat takes place in the same manner, and it is clear that the molecules in crystals, being regularly disposed with their axes in definite directions, will be electrified. Thermo-electrical currents are also explained. For if one of the junctions of a circuit consisting of two dissimilar metals is heated, the more sensitive metal will receive more electrical energy than the other, and give rise to a positive current. The potential difference at the junction will depend on the internal constants of the molecules in the two metals, so that we cannot expect to be able to express it by any simple general law.

(3) Electrification by simple contact of two dissimilar metals is not so easily explained if the effects of heat, pressure, and friction are excluded. It is, however, possible that the close contact of differently vibrating molecules may disturb the internal and therefore the external energy, and thus give rise to electrification. The electrification of similar metals by contact could be explained in the same way.

(4) Electrification by chemical action is completely explained by the author's theory, the production of electrical vibrations by this means being exactly analogous to the similar production of heat- and light-vibrations. Such chemical action must, in the author's opinion, play an important part in the galvanic cell, though contact electrification may also have a share in the action. The contact between copper and sulphuric acid, for example, is a very intimate one. At ordinary temperatures the molecules of both substances will be in motion. When two different molecules collide, their internal equilibrium will be destroyed, and they will therefore, according to § 8 (September 6, p. 460) form a chemical compound, provided the critical vibrations of the compound are, at the given temperature, less easily excited than those of the separate elements, which we must assume to be the case, from the strong chemical affinity which is experimentally known to exist between copper and sulphuric acid. During this process electrification will take place if the maximum internal electrical energy is less for the compound than for the constituents, exactly as hydrogen in combining with oxygen to form water produces light, and chlorine in combining with hydrogen to form hydric chloride produces heat. The electricity set free will be carried away by the copper, the latter being a good conductor. The accumulation of electricity in the copper is prevented, however, by its being used up again in forming a chemical compound with the zinc.

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(To be continued.)

COMPRESSIBILITY OF WATER, SALT-WATER, MERCURY, AND GLASS.¹

THE pressures employed in the experiments ranged from 150 to 450 atmospheres, so that results given below for higher or lower pressures [and inclosed in square brackets] are extrapolated.

¹ Extracted, with the sanction of Dr. Murray, from a Report by Prof. Tait, now in type for a forthcoming volume of the *Challenger* publications.

A similar remark applies to temperature, the range experimentally treated for water and for sea-water being only 0° to 15° C. Also it has been stated that the recording indices are liable to be washed down the tube, to a small extent, during the relief of pressure, so that the results given are probably a little too small.

Compressibility of mercury, per atmosphere 0.0000036
 ,, ,, gla-s 0.0000026

Average compressibility of fresh water per atmosphere—

[At low pressures	520 . 10 ⁻⁷ - 355 . 10 ⁻⁹ l + 3 . 10 ⁻⁹ l ²]
For 1 ton = 152.3 atm.	504 360 4
2 ,, = 304.6 ,,	490 365 5
3 ,, = 456.9 ,,	478 370 6

The term independent of *t* (the compressibility at 0° C.) is of the form—

$$10^{-7}(520 - 17p + p^2),$$

where the unit of *p* is 152.3 atmospheres (1 ton-weight per square inch). This must not be extended in application much beyond *p* = 3, for there is no warrant, experimental or other, for the minimum which it would give at *p* = 8.5.

The point of minimum compressibility of fresh water is probably about 60° C. at atmospheric pressure, but is lowered by increase of pressure.

As an approximation through the whole range of the experiments we have the formula—

$$\frac{0.00186}{36 + p} \left(1 - \frac{3t}{400} + \frac{t^2}{10000} \right);$$

while the following formula exactly represents the average of all the experimental results at each temperature and pressure—

$$10^{-7}(520 - 17p + p^2) - 10^{-9}(355 + 5p)t + 10^{-9}(3 + p)t^2.$$

Average compressibility of sea-water (about 0.92 of that of fresh water)—

[At low pressures	481 . 10 ⁻⁷ - 340 . 10 ⁻⁹ l + 3 . 10 ⁻⁹ l ²]
For 1 ton	462 320 4
2! ,,	447.5 305 5
3 ,,	437.5 295 5

Term independent of *t*—

$$10^{-7}(481 - 21.25p + 2.25p^2).$$

Approximate formula—

$$\frac{0.00179}{38 + p} \left(1 - \frac{t}{150} + \frac{t^2}{10000} \right).$$

Minimum compressibility point, probably about 56° C. at atmospheric pressure, is lowered by increase of pressure.

Average compressibility of solutions of NaCl for the first *p* tons of additional pressure at 0° C. :—

$$\frac{0.00186}{36 + p + s},$$

where *s* of NaCl is dissolved in 100 of water.

Note the remarkable resemblance between this and the formula for the average compressibility of fresh water at 0° C., and *p* + *s* tons of additional pressure.

[Various parts of the investigation seem to favour Laplace's view that there is a large molecular pressure in liquids. In the text it has been suggested, in accordance with a formula of the kinetic theory of gases, that in water this may amount to about 36 tons-weight on the square inch. In a similar way it would appear that the molecular pressure in salt solutions is greater than that in water by an amount directly proportional to the quantity of salt added.]

Six miles of sea, at 10° C. throughout, are reduced in depth 620 feet by compression. At 0° C. the amount would be about 663 feet, or a furlong. (This quantity varies nearly as the square of the depth.) Hence the pressure at a depth of 6 miles is nearly 1000 atmospheres.

The maximum-density point of water is lowered about 3° C. by 150 atmospheres of additional pressure.

From the heat developed by compression of water I obtained a lowering of 3° C. per ton-weight per square inch.

From the ratio of the volumes of water (under atmospheric pressure) at 0° C. and 4° C., given by Despretz, combined with my results as to the compressibility, I found 3°.17 C.; and by direct experiment (a modified form of that of Hope) 2°.7 C.

The circumstances of this experiment make it certain that the last result is too small.

Thus, at ordinary temperatures, the expansibility of water is increased by the application of pressure.

In consequence, the heat developed by sudden compression of water at temperatures above 4° C. increases in a higher ratio than the pressure applied; and water under 4° C. may be heated by the sudden application of sufficient pressure.

The maximum density coincides with the freezing-point at - 2°.4 C., under a pressure of 2.14 tons.

SCIENTIFIC SERIALS.

In the *Journal of Botany* for August and September, a considerable portion is occupied by the continuation of papers, to which reference has already been made—Messrs. Britten and Boulger's biographical index of British and Irish botanists, and Mr. G. Murray's catalogue of the marine Algae of the West Indian region.—Mr. W. H. Beeby records an addition to the British Phanerogamic flora in *Callitriche polymorpha*.—Mr. A. Fryer has some critical remarks on *Potamogeton fluitans*.—A number of new ferns from Western China, and from Manipur, in India, are described by Mr. J. G. Baker and Colonel Beddome.

THE numbers of the *Botanical Gazette* for June–August contain quite an unusual number of articles of general interest. Bryologists will find a description of eight new species of moss from North America, each illustrated by a plate; in fact, the plates in these three numbers are very numerous and excellent.—Mr. Chas. Robertson discusses the origin of zygomorphic flowers from the point of view of evolution.—Of flowering plants, we have descriptions of new species from Western America (chiefly Umbelliferae) and from Guatemala, by Messrs. Coulter and Rose and Mr. J. D. Smith.—Mr. F. C. Newcombe describes the mode of dissemination of the spores of Equisetum in the splitting of the sporangia and the carriage of the spores by means of the elaters.—Mr. A. F. Förste describes (with a plate) the adaptation to cross-fertilization in various species.

American Journal of Mathematics, 1888 (Baltimore, Johns Hopkins University).—The object of M. R. Liouville's paper, "Sur les lignes géodésiques des surfaces à courbure constante," with which vol. x. No. 4 opens, is stated by him to be "d'indiquer la signification géométrique des équations différentielles du second ordre ayant leur intégrale générale linéaire par rapport aux constantes arbitraires, et de former leurs invariants pour toutes les substitutions qui ne changent point, soit l'inconnue, soit la variable indépendante" (pp. 283–292).—The following memoir, on the primitive groups of transformations in space of four dimensions, by James M. Page, is likely to be very serviceable, as it is the first continuous account in English of the researches of Sophus Lie on the theory of groups of transformations. Lie himself has developed the theory in a series of papers which date from 1873, and has not published any connected work on the subject (pp. 293–346).—W. C. L. Gorton writes on line congruences. He treats the subject by quaternions, and obtains all Kummer's results (*Crelle*, vol. lvii.), and is enabled by his method to carry out certain steps which are only indicated by this writer (pp. 346–367).—The volume closes with a notelet by Prof. Franklin, entitled "Some Theorems concerning the Centre of Gravity." This contains "almost instantaneous" proofs of Lagrange's two theorems on the centre of gravity.

With vol. xi. No. 1, we have what strikes us as being an admirable likeness of the great French mathematician, Charles Hermite. We have previously expressed our pleasure at this new departure of the editors of this journal, and hope their catering for mathematicians will meet with material approval.—The first communication is a memoir on a new theory of symmetric functions, by Captain P. A. Macmahon, R.A. This prolific young mathematician is doing excellent work, and the pages of the journal are just suited to present his results in the most effective form. The paper is intimately connected with a recent one, by the same writer, communicated to the London Mathematical Society, in which he gives a sketch of an extension of the algebra of the theory of symmetrical functions, and establishes the basis of a wide development. "The main object of the memoir is to show clearly