picture invariably declines. The physical type, too, of the Babylonian statues from Tel-lo, approaches the Caucasian rather than the Semitic type.

ON SOME RECENT AMERICAN MATHEMATICAL TEXT-BOOKS

IN NATURE (vol. xvi. p. 21) we drew attention to a "shaking" that was taking place among the "dry bones" of the mathematical text-books in common use in American colleges and schools, and upon the analysis we then furnished of a few works before us we ventured to predict a speedy awakening of mathematical life. Our prognostications have been quickly fulfilled, and we now propose to submit an account of five recent books, some of which are quite fitted to hold their own, in our opinion, with English text-books on the same subjects.

"The Elements of the Integral Calculus, with a Key to the Solution of Differential Equations," by Dr. W. E. Byerly (Boston, 1881), is a sequel to the volume on the "Differential Calculus," previously noticed by us. This work is founded upon Bertrand's classical treatise, supplemented by free use of the allied treatises by Todhunter, Boole, and Benjamin Peirce. The opening chapters give a clear exposition of the use of symbols of operation and of imaginaries. So early an introduction to these subjects is novel to us in this connection, but it shows how the subject of quaternions is coming to the front, and the passage from the subjects of these chapters to quaternions is but a short one The main portion of the book calls for no special comment. In Chapter XIV. we have a treatment of *mean value and probability*, founded upon the able contributions of Prof. M. W. Crofton, F.R.S., to Mr. Williamson's treatise.

The novelty of the book is Chapter XV., entitled "Key to the Solution of Differential Equations." This key is based upon Boole's work, and is a collection of concise, practical rules for the solution of these equations. An idea of its form will be best conveyed to some persons by saying that it resembles the analytical key so frequently prefixed now-a-days to handbooks of the British (and other) flora. By a series of references we run the particular equation to ground. Thus, taking the example, $(1+x)y\,dx+(1-y)x\,dy=0$, it is a single equation, this sends us to a number; it involves ordinary derivatives, this advances us a stage; it contains two variables, is of the first order, and finally of the first degree. The upshot is we arrive at the form $X\,dx+Y\,dy=0$, under which head we learn how to solve the equation. Under this last head, as throughout the book, are given numerous illustrative exercises for practice.

Dr. A. S. Hardy's "Elements of Quaternions" (Boston, 1881) is intended to meet the wants of beginners. In addition to the works of Sir William R. Hamilton and Prof. Tait, the author has consulted the memoirs or works of Bellavitis ("Calcolo dei Quaternione" and the "Exposition de la Méthode des Équipollences" in Laisant's translation); Hoüel's "Quantités Complexes;" Argand's "Essai" (1806); Laisant's "Applications mécaniques du Calcul des Quaternions," and one or two other books and papers in the *American Journal of Mathematics*, vol. i. p. 379. It is a good introduction to such a work as Prof. Tait's, the originality and conciseness of which, however, Dr. Hardy thinks to be "beyond the time and need of the beginner."

Our next book is "An Elementary Treatise on Mensuration," by G. B. Halsted (Boston, 1881). Dr. Halsted is already known to mathematicians here as the author of a very full "Bibliography of Hyper-space and non-Euclidean Geometry," in the *American Journal of Mathematics*, vol. i., Nos. 3, 4. This treatise on Metrical Geometry is "the outcome of work on the subject while teaching it to large classes," so that it is no hastily prepared book, but has been founded on actual teaching experience. The methods have a German "smell," and this is justified by the author's residence, we presume as a student, at Berlin. There are eight chapters: (1) on the measurement of lines (triangles, method of limits, rectification of the circle; (2) on the measurement of angles; (3) of plane areas; (4) of surfaces (he uses *Mantel* for lateral surfaces, also *Steregon* and *Steradian* in connection with a solid angle); (5) of volumes (*Quader* is new for De Morgan's "right solid"). In these last two chapters the solids discussed are the prism, cylinder, pyramid, cone, and sphere; an article is also devoted to Pappus's theorem. (6) The applicability of the prismoidal formula; (7) approximative methods, as Weddle's method; (8) on the mass-centre, with a paragraph on the mass-centre of an octahedron, which gives Clifford's construction (see *Proc.* Lond. Math. Soc., vol. ix. p. 28). There are numerous exercises, these we have not tested. The book is most effectively "got up," the printing, figures, and paper being, to our mind, excellent.

Our last two books are by Prof. Simon Newcomb, so well known as the author of "Popular Astronomy." The first, "Algebra for Schools and Colleges" (New York, 1881), has already reached its second edition. It is a capital book; indeed we are disposed to rank it as the best manual on the subject that we have seen for school purposes. It is divided into two portions, "the first adapted to well-prepared beginners, and comprising about what is commonly required for admission to colleges, and the second designed for the more advanced general student." We shall perhaps best serve the end we have in view in noticing this work by giving an analysis of the author's preface. The principles of construction are (1) that an idea cannot be fully grasped by the youthful mind unless it is presented in a concrete form. Hence numerical examples of nearly all algebraic operations and theorems are given—so numbers are preferred to literal symbols in many cases. The relations of positive and negative algebraic quantities are represented by lines and directions at the very earliest stage. "Should it appear to any one that we thus detract from the generality of algebraic quantities, it is sufficient to reply that the system is the same which mathematicians use to assist their conceptions of advanced algebra, and without which they would never have been able to grasp the complicated relations of imaginary quantities." Principle (2) is that all mathematical conceptions require time to become engrafted upon the mind, and the longer, the abstruser they are. "It is from a failure to take account of this fact, rather than from any inherent defect in the minds of our youth, that we are to attribute the backward state of mathematical instruction in this country, as compared with the continent of Europe." Prof. Newcomb considers the true method of meeting this difficulty is to adopt the French and German plan of teaching algebra in a broader way, and of introducing the more advanced conceptions at the earliest practicable period in the course. A third feature is the minute subdivision of each subject, and the exercising the pupil on the details before combining them inta a whole. This remark especially applies to the solution of the exercises. Some subjects have been omitted (as G.C.D. of polynomials, square roots of binomial surds, and Sturm's theorem), as they have no application "in the usual course of mathematical study, nor advance the student's conception of algebra," and in studying them there is a waste of power. "Thoroughness" has been our author's aim rather than "multiplicity of subjects." There are other points of interest in this preface which show that the writer is a very experienced teacher, and which we commend to the consideration of teachers here, but we must pass on to indicate the contents of the two parts.

Part I. embraces algebraic language and operations, equations, ratios and proportion, powers and roots, equations (quadratic), progressions, seven books in all.

Part II. treats of relations between algebraic quantities (functions, &c.), the theory of numbers (also continued fractions), the combinatory analysis (including probabilities), series and the doctrine of limits, imaginary quantities (operations with the imaginary unit and the geometrical representation of imaginary quantities: note our remarks above on this head under Byerly), the general theory of equations.

The second of Prof. Newcomb's works before us is "Elements of Geometry" (New York, 1881). An article in our columns (NATURE, vol. xxi. p. 293), headed "The Fundamental Definitions and Propositions of Geometry, with especial Reference to the 'Syllabus' of the Association for the Improvement of Geometrical Teaching," gives its readers a hint that some such work as the one before us was even then in the author's mind—"A summary of my own, the latter [*i.e.* the summary] still in an inchoate state." The remarks in this article showed that their writer was well fitted to address himself to the subject of a geometrical text-book, and the execution is not at all inferior to the promise. The ground taken up is the Euclidian geometry as comprised in the treatises of Euclid himself, Legendre, and Chauvenet. As with the "Algebra," here let Prof. Newcomb speak for himself. As he himself says, the question of the best form of development is one of great interest at the present time among both teachers and thinkers. The object not being to teach geometry merely, but the general training of the powers of thought and expression being a main object, Prof. Newcomb considers it most important to guard against habits of loose thought and incomplete expression to which the pupil is prone. This he considers is best secured by teaching the subject on the old lines. The defects he finds in Euclid's system are (1) in the treatment of angular magnitude; here he makes two additions, the explicit definition of the angle which is equal to the sum of two right angles, and the recognition of the sum of two right angles as itself an angle. He adopts, from the "Syllabus," the term "straight angle," though he himself inclined (NATURE, *loc. cit.*) to the use of "flat angle," and considers the German "gestreckte Winkel" to be more expressive. Then (2) the restriction of the definition of plane figures to portions of a plane surface. "In modern geometry figures are considered from a much more general point of view as forms of any kind, whether made up of points, lines, surfaces, or solids." In an appendix, "Notes on the Fundamental Concepts of Geometry" he returns to a consideration of this subject.

Features of the book are (1) the practising the student in the analysis of geometrical relations by means of the eye before instructing him in formal demonstrations; (2) the application of the symmetric properties of figures in demonstrating the fundamental theorem of parallels (*cf.* German methods and Henrici's conguent figures); (3) the analysis of the problems of construction, to lead the pupil to discover the construction himself by reasoning; (4) the division of each demonstration into separate numbered steps, and the statement of each conclusion, where practicable, as a relation between magnitudes; (5) the theorems for exercise have been selected with a view to interesting the student in the study, and the author has endeavoured to graduate them in order of difficulty; (6) some of the first principles of conic sections have been unfolded, more especially for the use of students who do not propose to study analytical treatises on those curves; (7) Euclid's treatment of proportion is "perfectly rigorous, but has the great disadvantages of intolerable prolixity, unfamiliar conceptions, and the non-use of numbers. The system common in American works of treating the subject merely as the algebra of fractions, has the advantage of ease and simplicity." But to this last system there are obvious objections, and our author essays with some reserve, a *via media*. In this part and in the following Prof. Newcomb submits his methods to the judgment of teachers. Feature (8) involves the treatment of the fundamental relations of lines and planes in space. "In presenting it he has been led to follow more closely the line of thought in Euclid than that in modern works. At the same time he is not fully satisfied with his treatment, and conceives that improvements are yet to be made."

It will be gathered that the book covers most of the ground passed over by young students in plane and solid geometry, and conics in their school training. We cordially commend both Prof. Newcomb's works to teachers in this country, and we feel sure they will not regret our having called their attention to them so fully in the author's own words, as they will thus see in what way his books are likely to be helpful to them. We have read them with much interest, and feel sure our readers will endorse our favourable verdict upon them. We need only say that the author considers that the study of geometry as here unfolded can be advantageously commenced at the age of twelve or thirteen years. The volumes, with a third, which we have not seen, on Astronomy, form part of "Newcomb's Mathematical Course."

R. TUCKER

ELECTRICITY AT THE CRYSTAL PALACE

IV.—*Submarine Telegraphy*

IN the stall of the South Eastern Railway Company at the Crystal Palace may be seen a specimen of the first cable core ever submerged. It consists of a slender copper wire coated with gutta-percha, and was prepared at Streatham by Mr. J. Forster. On January 10, 1849, it was submerged by Mr. Walker, at Folkestone, and a copy of the telegram announcing the completion of the work is still preserved. It runs: "I am on board the *Princess Clementine*. I am successful; 12.49 p.m." Next year a cable was laid between Dover and Cape Grisnez by Mr. Wollaston, but lasted only a few hours. Several specimens of it are shown in the Exhibition by the South Eastern Railway Company and the Post Office. The gutta-percha core was quite unprotected, and it was sunk by means of lead weights attached at intervals. Next year a core, protected by hemp and iron sheathing, was laid by Mr. T. R. Crampton between Dover and Cape Grisnez, and proved so successful, that it is still working. Specimens of this cable, which has proved the type of all subsequent ones, are also to be seen.

There are now some 97,200 miles of cable at work in the world, and before this year is ended the hundred thousand miles will have been attained; for the second Jay Gould Atlantic cable is still unfinished, and the s.s. *Silvertown* of the India-rubber and Gutta-percha Telegraph Company is now on her way to lay some two thousand miles on the West Coast of Central America. Nearly all this cable has been made in London, and the Telegraph Construction and Maintenance Company alone has manufactured 65,400 miles, and laid it in almost every sea, in depths varying from shoal water to 3000 fathoms. In 1863 the firm was resolved into the existing Company. Specimens of all the cables made by them are exhibited in a large glass case, together with a large map of the world, showing all the submarine and land lines in existence; those constructed by the Company being marked in red. The most novel feature of their exhibit is, however, a plan for keeping up telegraphic communication between a lightship and the shore. In 1870 an attempt was made to establish a floating telegraph station in the chops of the Channel; an old man-of-war corvette, the *Brisk*, being fitted up, and moored in deep water about sixty miles from the Land's End. It was found, however, that as the ship swung with the tide, the telegraph cable fouled with the ship's riding-chain, and likewise became twisted into kinks, which crushed