THURSDAY, FEBRUARY 9, 1871

## THE POWER OF NUMERICAL DISCRIMINATION

IT is well known that the mind is unable through the eye to estimate any large number of objects without counting them successively. A small number, for instance three or four, it can certainly comprehend and count by an instantaneous and apparently single act of mental attention. The limits of this power have been the subject of speculation or experiment among psychologists, and Sir William Hamilton thus sums up almost the whole of what is known about it :—

"Supposing that the mind is not limited to the simultaneous consideration of a single object, a question arises, How many objects can it embrace at once? . . . . I find this problem stated and differently answered by different philosophers, and apparently without a knowledge of each other. By Charles Bonnet, the mind is allowed to have a distinct notion of six objects at once; by Abraham Tucker the number is limited to four; while Destutt Tracy again amplifies it to six. The opinion of the first and last of these philosophers appears to me correct. You can easily make the experiment for yourselves, but you must beware of grouping the objects into classes. If you throw a handful of marbles on the floor, you will find it difficult to view at once more than six, or seven at most, without confusion; but if you group them into twos, or threes, or fives, you can comprehend as many groups as you can units, because the mind considers these groups only as units; it views them as wholes, and throws their parts out of consideration. You may perform the experiment also by an act of imagination." (Lectures, vol. i. pp. 253-4)

This subject seemed to me worthy of more systematic investigation, and it is one of the very few points in psychology which can, as far as we yet see, be submitted to experiment. I have not found it possible to decide conclusively in the manner Hamilton suggests, whether 4 or 5 or 6 is the limit, nor do imaginative acts of experiment seem likely to advance exact knowledge. Probably the limit is not really a definite one, and it is almost sure to vary somewhat in different individuals.

I have investigated the power in my own case in the following manner. A round paper box $4\frac{1}{2}$ inches in diameter, lined with white paper, and with the edges cut down so as to stand only $\frac{1}{4}$ inch high, was placed in the middle of a black tray. A quantity of uniform black beans was then obtained, and a number of them being taken up casually were thrown towards the box so that a wholly uncertain number fell into it. At the very moment when the beans came to rest, their number was estimated without the least hesitation, and then recorded together with the real number obtained by deliberate counting. The whole value of the experiment turns upon the rapidity of the estimation, for if we can really count five or six by a single mental act, we ought to be able to do it unerringly at the first momentary glance.

Excluding a few trials which were consciously bad, and some in which the number of beans was more than 15, I made altogether 1,027 trials, and the following table contains the complete results :—

| Estimated Numbers. | Actual Numbers. | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 3 | 23 | | | | | | | | | | | | |
| 4 | | 65 | | | | | | | | | | | |
| 5 | | | 102 | 7 | | | | | | | | | |
| 6 | | | 4 | 120 | 18 | | | | | | | | |
| 7 | | | 1 | 20 | 113 | 30 | 2 | | | | | | |
| 8 | | | | | 25 | 76 | 24 | 6 | 1 | | | | |
| 9 | | | | | | 28 | 76 | 37 | 11 | 1 | | | |
| 10 | | | | | | 1 | 18 | 46 | 19 | 4 | | | |
| 11 | | | | | | | 2 | 16 | 26 | 17 | 7 | 2 | |
| 12 | | | | | | | | 2 | 12 | 19 | 11 | 3 | 2 |
| 13 | | | | | | | | | | 3 | 6 | 3 | 1 |
| 14 | | | | | | | | | | 1 | 1 | 4 | 6 |
| 15 | | | | | | | | | | | 1 | 2 | 2 |
| Totals . . | 23 | 65 | 107 | 147 | 156 | 135 | 122 | 107 | 69 | 45 | 26 | 14 | 11 |

The above table gives the number of trials in which each real number was correctly or incorrectly guessed; thus in 120 cases 6 was correctly guessed; in 7 cases it was mistaken for 5, and in 20 for 7. So far as my trials went, there was absolute freedom from error in the numbers 3 and 4, as might have been expected; but I was surprised to find that several times I fell into error as regards 5, which was wrongly guessed in 5 per cent. of the cases. Abraham Tucker thus appears more correct as to my power than the other philosophers.

But in reality the question is not to be so surely decided by the trial of the few first numbers, as by endeavouring to obtain some general law pervading the whole series of trials. Calculating the average error of estimation in the case of each number, without regard to the direction of the error, we get the following numbers :—

| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | ·06 | ·18 | ·27 | ·44 | ·41 | ·65 | ·81 | ·73 | 1·08 | 1·21 | 1·27 |

These numbers vary pretty regularly in an apparently linear manner, except that in the case of the numbers 9 and 12, the result is too small. The error is simply proportional to the excess of the real number over $4\frac{1}{2}$, or obeys a law expressed in the formula ($n$ being the real number)—

$$\text{error} = m \times (n - 4\tfrac{1}{2})$$

When we calculate the constant $m$ for each number it comes as follows :—

| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|
| ·112 | ·122 | ·110 | ·127 | ·091 | ·117 | ·125 | ·098 | ·127 | ·128 | ·121 |

These numbers are sufficiently equal to enable us to take the average 0·116 as a good result, and the formula then becomes—

$$\text{error} = 0{\cdot}116 \times (n - 4\tfrac{1}{2})$$

or approximately—

$$\text{error} = \frac{n}{9} - \tfrac{1}{2}$$

This is a purely empirical law, the meaning or value of which I cannot undertake to explain. The most curious point is that it seems to confirm my previous conclusion that my own power of estimating the number *five* is not perfect. The limit of complete accuracy, if there were one, would be neither at 4 nor 5, but half-way between them; but this is a result as puzzling as one of the uninterpretable symbols in mathematics, just, for instance, like the factorial of a fractional number. But I give it for what it may be worth.

When we take into account the direction of the errors, the results are as follows:—

| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|
| +·06 | +·09 | +·05 | 0·0 | −·05 | −·27 | −·46 | −·51 | −·85 | −·93 | −1·27 |

Thus there is a clear tendency to over-estimate small numbers and to under-estimate large ones. There is an evident inclination towards those medium numbers which most frequently recurred: how far this discredits the experiments I cannot undertake to say, but it is an instance of that inevitable bias in mental experiments against which it is impossible to take complete precautions.

My conclusion that the number five is beyond the limit of perfect discrimination, by some persons at least, is strongly supported by the principles of rhythm. All the kinds of time employed by musicians depend upon a division of the bar into two or three equal parts, or into multiples of these. Music has, indeed, been composed with the bar divided into five equal parts, but no musicians have yet been found capable of performing it (Rees' Cyclopædia, RHYTHM). Short runs, indeed, consisting of five or even seven equal notes, are not unfrequently employed by the best musicians, but it is to be doubted whether the ear can grasp them surely. I presume it is beyond doubt that 6, 8, 9, or more equal notes in a bar are always broken up by the hearer, if not by the performer, into periods of 2, 3, or 4. Quinary music, even if it could be executed, would be ill appreciated by the hearers, and, though all the powers of the human mind may be expected to progress in the course of ages, quinary rhythm belongs to the music of the distant future.

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## *BURMEISTER'S FAUNA ARGENTINA*

FEW districts of the world are so rich in well-preserved remains of an extinct fauna of remarkable and interesting character as the neighbourhood of the city of Buenos Ayres. The immense alluvial plain of the Argentine Republic is the burial-place of the Megatherium, the Mylodon, the Glyptodon, the Macrauchenia, the Toxodon, and many other strange forms of ancient life, whose bones are ever and anon restored to light by the crumbling away of the soft banks of the great rivers which flow into the estuary of the Plata. So abundant, indeed, are they that, as remarked years ago by Darwin, any line whatever drawn across the Pampas would probably cross the skeleton of some extinct animal.

Collections of these fossils have at various times been sent to several European museums, and much information has been published upon the nature of the animals to which they belonged, but these observations have been generally made upon imperfect or fragmentary materials. The fortunate circumstance of the able and energetic German naturalist, Dr. Hermann Burmeister, formerly Professor of Zoology in the University of Halle, having taken up his residence at Buenos Ayres, and having been appointed Director of the Public Museum of that city, has been the occasion of a systematic and elaborate elucidation of the ancient fauna of this important district.

This has been brought about mainly by the publication of a richly illustrated serial quarto work, entitled "Anales del Museo Publico de Buenos Aires," the special object of which is to describe and figure the new or little-known objects preserved in that establishment. This work, which was commenced in 1864, appears at irregular intervals, but has already reached its sixth number, the first five of which constitute the first volume, the sixth, which is just published, being the commencement of a second volume. Dr. Burmeister is the sole author, and we have no hesitation in saying that it promises to be one of the most important contributions yet made to the knowledge of Mammalian zoology, for to this class is the publication mainly restricted. The parts already before us contain not only far more complete descriptions and detailed figures than have hitherto been given, of many of the extinct forms mentioned above, but it has also several admirable anatomical memoirs on rare or little-known living forms, especially of the Cetacea which occur in the estuary of the great river Plata, and in the adjoining part of the Atlantic Ocean, a field of research hitherto almost unexplored.

As this work, being written in the Spanish language, is not so well known in this country as it deserves to be, we propose to lay before our readers a summary of the contents of the volume already completed, from which they will be able to judge of the richness and variety of the material which has been at the author's disposal, and of the excellent use that has been made of it in his experienced hands.

After a history of the foundation and progress of the public museum of Buenos Ayres, and a general essay on palæontology, a detailed description is given of the skeleton of *Macrauchenia patachonica*. The first discovered remains of this very remarkable animal, which is about the size of a camel, were found by Mr. Darwin in 1834 at Port St. Julian on the Patagonian coast, and presented by him to the Museum of the Royal College of Surgeons of London, where they are now preserved. They were described by Professor Owen in the appendix to the "Voyage of the *Beagle*" (1840). Since that time but little addition was made to our knowledge of the species (although some bones of a smaller animal of the same genus, discovered by Mr. D. Forbes in Bolivian copper mines, have been described by Professor Huxley), until the lamented Bravard commenced the description, in a work to be entitled "Fauna fósil del Plata," of a comparatively perfect skeleton, which was contained in the museum of Buenos Ayres; but as he did not recognise its identity with Owen's Macrauchenia, he gave it the name of *Opisthorhinus falconeri*. The premature death of Bravard in the earthquake which destroyed the greater part of the town of Mendoza, prevented the publication of this work; but three of the plates, which had already been executed, containing figures of the skull with nearly complete dentition, and many of the vertebræ and limb bones, form the first three plates of the present work. To these, Dr. Burmeister has added another containing views of the pelvis and some more vertebræ, and an elaborate description of the whole of the known bones, finally concluding that the zoological position of the genus is among the imparidigitate or perissodactyle Ungulata, between the Horse and the Tapir.

After some remarks on the humming-birds described by Azara, a preliminary notice is next given on the different species of Glyptodon, or gigantic extinct Armadillo, in the museum. Three species are distinguished as well esta-