Supplementary Methods

Optimal pooling rules

We note that the Optimal pooling rule relies on the covariance matrix of responses at different sites. If we let \( x_i \) be the observed amplitude at site \( i \) \((i = 1, \ldots, n)\) on trial \( j \) \((j = 1, \ldots, m)\), then the covariance matrix \( \Sigma \) is an \( n \times n \) matrix, where \( \Sigma(i,i) \) is the variance of \( x_i \) across the \( m \) trials, and \( \Sigma(i,k) \) is the covariance of \( x_i \) and \( x_k \) across the \( m \) trials.

The optimal pooling rule for two sites is easy to apply and provides useful intuitions about how correlated noisy responses should be combined (see Fig. 3 and Supplementary Fig. 6). Consider responses from a pair of sites with sensitivity \( \langle d_1', d_2' \rangle \), standard deviations \( \langle \sigma_1, \sigma_2 \rangle \) and correlation \( r \). Applying Eq. Error! Reference source not found., we obtain the optimal set of weights:

\[
(1) \quad w = \langle w_1, w_2 \rangle = \left\langle \frac{d_1' - rd_2'}{\sigma_1 (1 - r^2)}, \frac{d_2' - rd_1'}{\sigma_2 (1 - r^2)} \right\rangle
\]

Further, applying Eq. (3), the combined sensitivity at the two sites is given by

\[
(2) \quad d_{\text{pooled}}' = \sqrt{\left(\frac{d_1'}{\sigma_1 (1 - r^2)}\right)^2 + \frac{1}{1 - r^2} \left(\frac{d_2' - rd_1'}{\sigma_2 (1 - r^2)}\right)^2}
\]

Supplementary Fig. 6 shows \( w_2 \) and \( d_{\text{pooled}}' \) computed using Equations (4) and (5) in the example experiment.

The basic result for optimal pooling over two sites applies in the more general case where responses are pooled from \( n \) sites. However, there are often practical difficulties in determining the inverse covariance matrices required by equation (2); for
example, it is often impossible to invert very large matrices. A standard way to get around these difficulties is to determine the optimal weights using Fourier methods. Specifically, the Fourier transform of the average 2D spatial correlation function (the radial 2D version of Fig. 2f) gives the power spectrum of the Gaussian noise. From the power spectrum of the noise we can compute a whitening kernel (a spatial filter), which, when convolved with the spatial response, will decorrelate the noise in the response. Fig. 3c shows a cross section of the whitening kernel for the example experiment. It is well known that the inverse covariance matrix $\Sigma^{-1} = \Lambda^T \Lambda$, where $\Lambda$ is the whitening matrix, and thus, convolving the whitening filter twice with the average response $s$ is mathematically equivalent to computing the optimal weights in Eq. (2) $^{38}$.

The detection sensitivity of the different pooling rules was determined using a jackknife procedure $^{40}$. In this procedure, a separate analysis is performed for each of the $m$ trials. For each trial, model parameters and pooled responses are computed for the remaining $m-1$ trials, and an optimal criterion is established based on those trials. This criterion is then applied to the pooled response from the unseen trial; the performance of the model is classified as correct, if the pooled response exceeds the criterion on a target-present trial, or remains below the criterion on a target-absent trial. This procedure is repeated for each trial in our data set to obtain the neurometric function. Analysis of simulated data shows that for the number of trials in our experiments (typically 100 trials), the jackknife procedure underestimates the detection sensitivity of the optimal pooling rule. This is an additional reason why performance estimates for the Optimal rule (Fig. 4 and Fig. 5) should be viewed as a lower bound on the actual neuronal sensitivity.
Correlations in neural populations

Exceedingly weak pairwise correlations between neurons can lead to high correlations between pools of neurons. Considering the simple case of two populations of neurons with uniform pairwise correlation within and between pools, the correlation \( R_N \) between the pooled (summed) responses is given by:

\[
R_N = \frac{N r}{1 + (N - 1) r}
\]

where \( N \) is the number of neurons in each pool and \( r \) is the pairwise correlation between individual neurons. This equation shows that even for very low pairwise correlations, the correlation between the pooled responses can reach high values for sufficiently large \( N \) (Supplementary Fig. 4).